Secure Multiparty Computation

and Team Matching

Shamir Secret Sharing

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n shares are then computed as: $\forall i \in \{1, \ldots, n\} : a_i = A(i), \quad b_i = B(i)$

A() and *B()* are degree *t* polynomials and can be uniquely defined by *n=t+1* points

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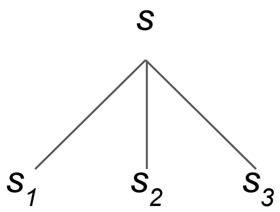

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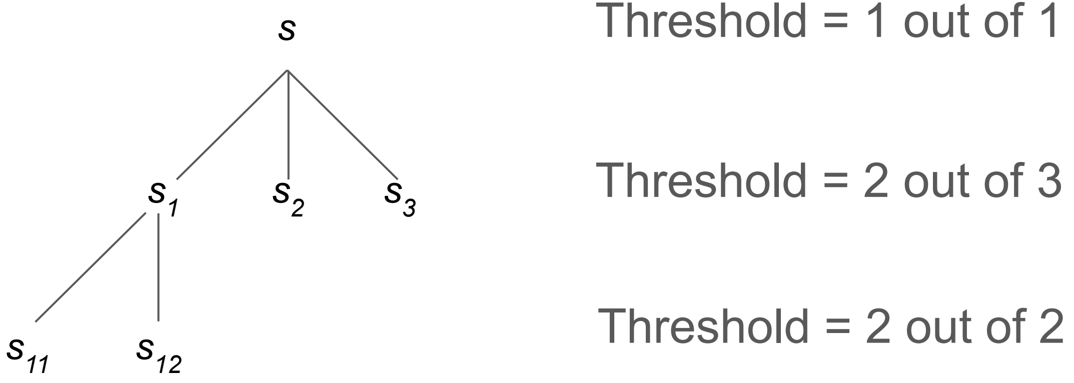

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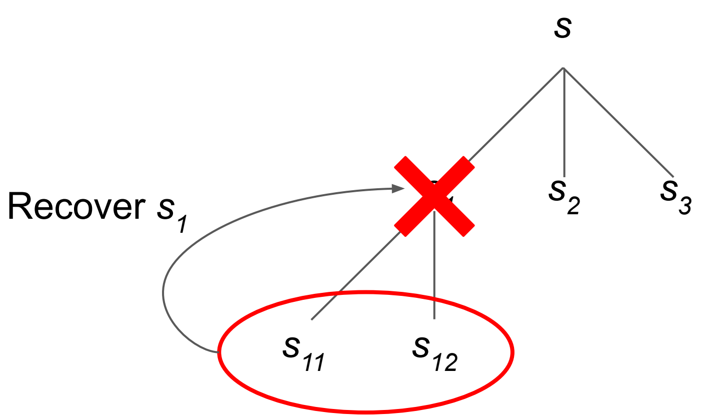

Threshold = 1 out of 1

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Threshold = 2 out of 2

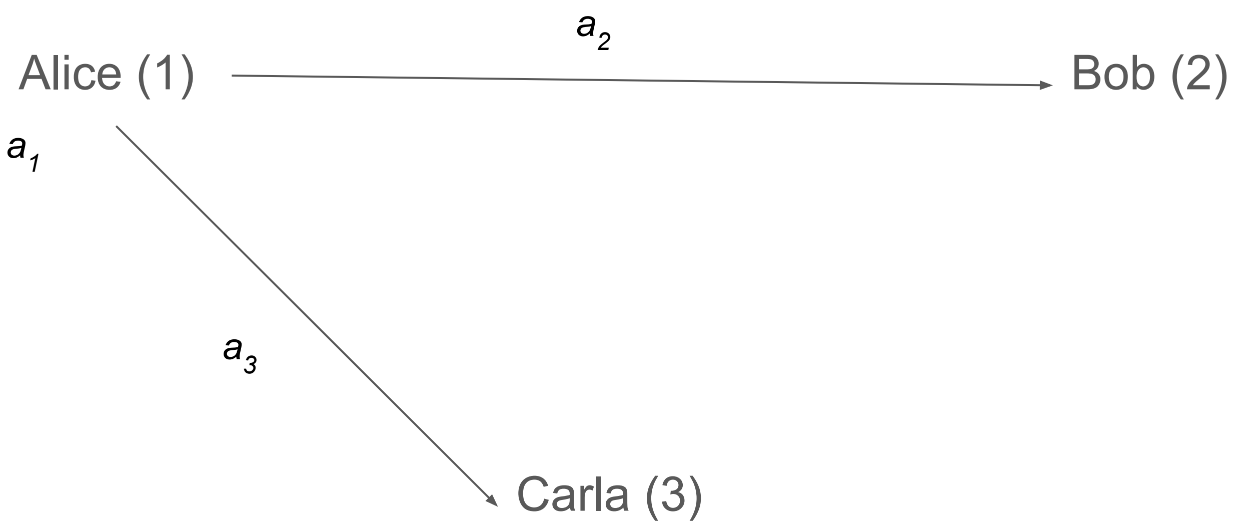
Secure Multiparty Computation (BGW)

Alice, Bob, and Carla all have a secret value.

They want to learn the output of some function on their secret inputs, but they **don’t** want the others to learn their input!

Canonical example of this is salary: several companies may want to learn what the average salary is for a role, but they want to keep their payroll information private.

Secure Multiparty Computation (yay)



Each party sends Shamir Secret Shares of their input to the other parties

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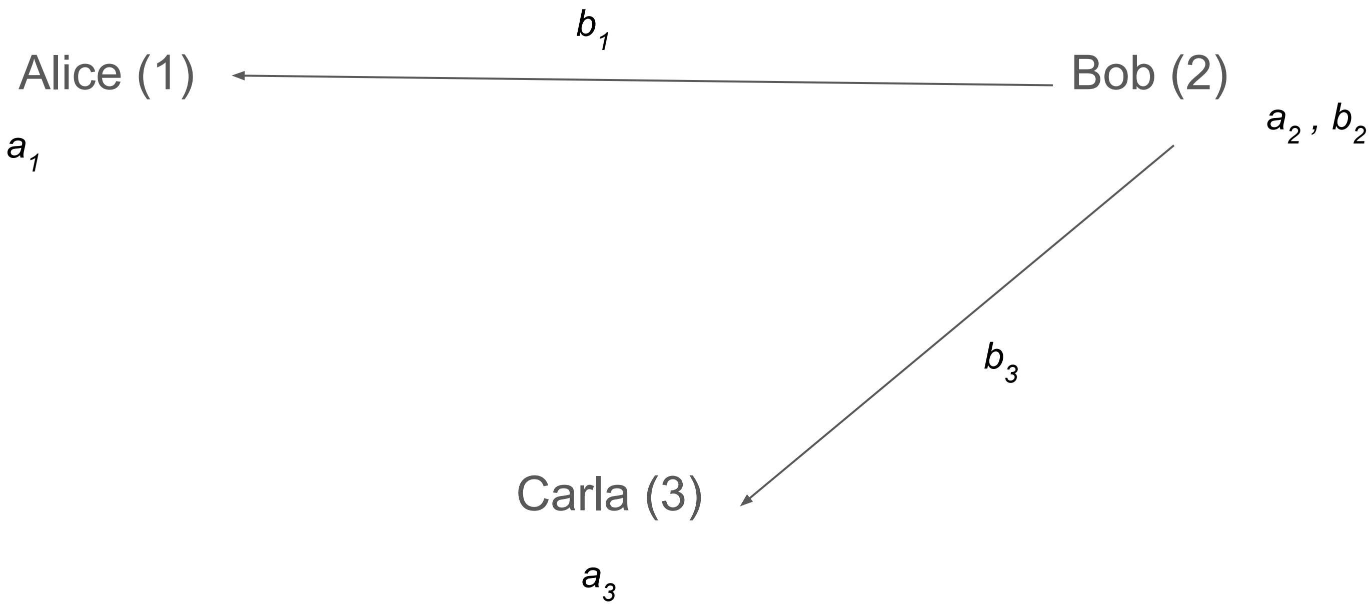

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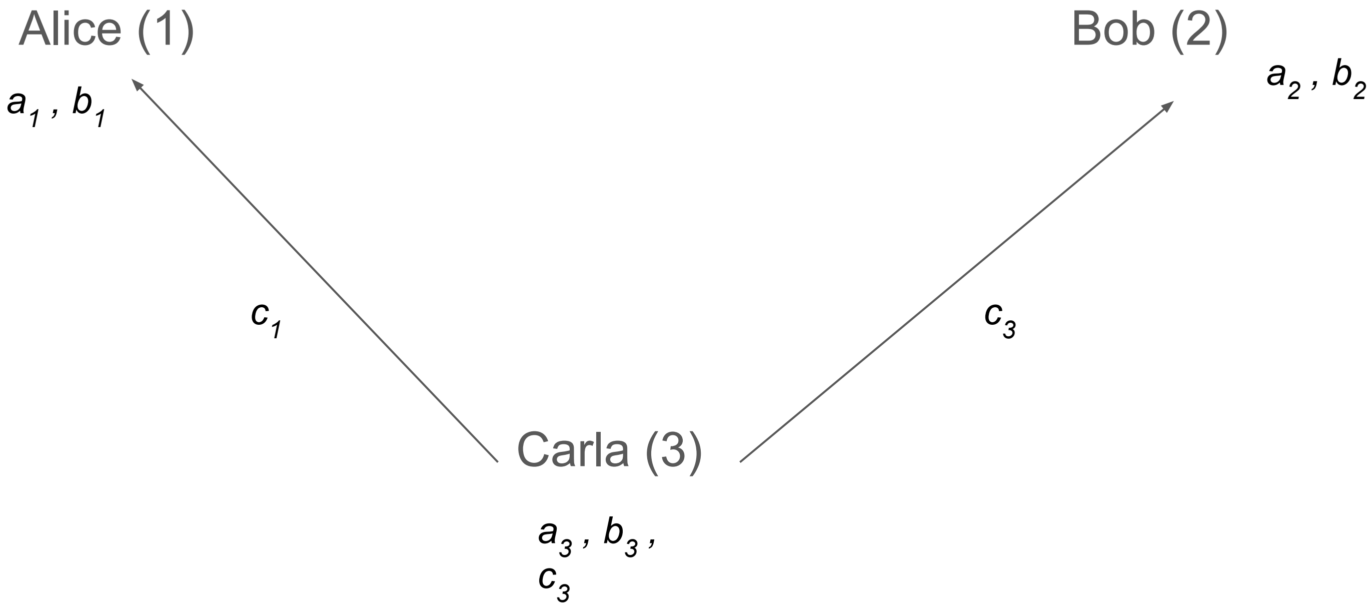

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Secure Multiparty Computation (yay)

Alice (1)

a_1 , b_1 ,
c_1

Bob (2)

a_2 , b_2 , c_2

Now what?

Carla (3)

a_3 , b_3 ,
c_3

Secure Multiparty Computation (yay)

Alice (1)

a_1 , b_1 ,
c_1

Bob (2)

a_2 , b_2 , c_2

They want to compute
f(a,b,c) = avg(a,b,c) =
sum(a,b,c)/3

Carla (3)

a_3 , b_3 ,
c_3

Secure Multiparty Computation (yay)

Alice (1)

a_1 , b_1 ,
c_1

Bob (2)

a_2 , b_2 , c_2

How to compute
a+b+c?

Carla (3)

a_3 , b_3 ,
c_3

Secure Multiparty Computation (yay)

Alice (1)

$sum(a,b,c)_1 = a_1 + b_1 + c_1$

Bob (2)

$sum(a,b,c)_2 = a_2 + b_2 + c_2$

Add shares locally!

Carla (3)

$sum(a,b,c)_3 = a_3 + b_3 + c_3$

Secure Multiparty Computation (yay)

Alice's secret is A(0) where A() = $\alpha_0 x^0 + \alpha_1 x^1 + \alpha_2 x^2$ (α_1 and α_2 are random! α_0 = a)
(need 3 points to define a degree 2 polynomial: Alice, Bob, and Carla each have one point

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B() = $\beta_0 x^0 + \beta_1 x^1 + \beta_2 x^2$ (β_1 and β_2 are random! β_0 = b)

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A() + B() + C() = D() where D(0) = A(0) + B(0) + C(0) (!!)

When Alice, Bob, and Carla add their shares locally they obtain a share (a point) of D() that can later be interpolated to learn D() and evaluate D(0) to learn the sum of their secret inputs!

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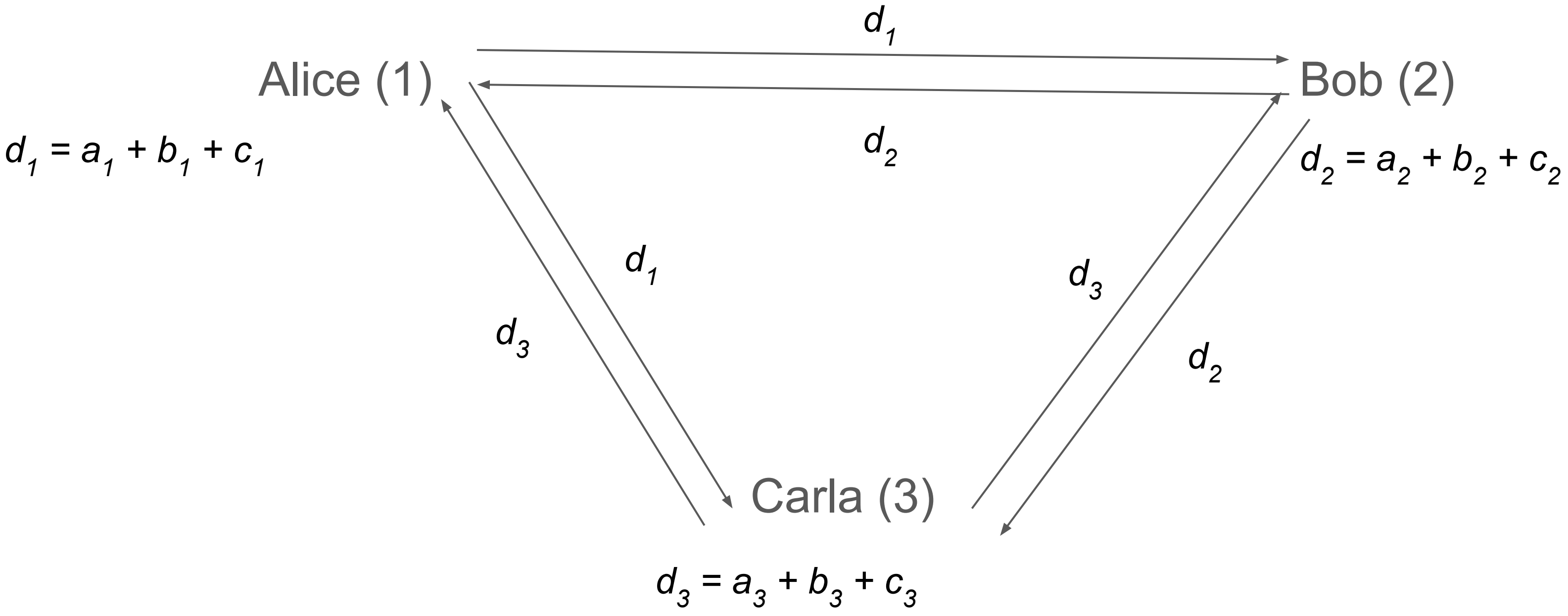

Secure Multiparty Computation (yay)

Alice (1)

Recover: $d = a + b + c$
Divide d by 3 to get average salary

Bob (2)

Recover: $d = a + b + c$
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Carla (3)

Recover: $d = a + b + c$
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MPC security model

Alice, Bob, and Carla learn *nothing* from participating in the protocol that could not have also been learned from only their own input and the protocol output.

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No point in computing a sum in 2pc (you'd learn the other party's input!)

Multiplication in MPC

Multiplication by a constant *c* can be done locally the same way as addition: multiply the shares α_n by *c* to get a share of a polynomial D() where D(0) = $c\alpha_0$

Multiplication of secrets is harder.

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Firstly, this will raise the degree of the polynomial!

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Also, A() * B() will not necessarily be a random polynomial.

Degree reduction and rerandomization is an interactive process that requires communication between Alice, Bob, and Carla.