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DATE: 2/22/17

FILE UNDER: Hash Functions II

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**Admin:**

Pset #1 due Mon 2/27
Pset #2 due Mon 2/27 (new groups)
Projects!

**Today:**

Cryptographic Hash Functions II ("Merkle Day")

- Merkle Trees
- Puzzles
- PK crypto based on puzzles ("Merkle puzzles")
- Constructions:
  - Merkle-Damgard
  - Keccak (SHA-3)

**Readings:**

Katz & Lindell: Chapter 5

Paar & Pelzl: Chapter 11

Ferguson: Chapter 5

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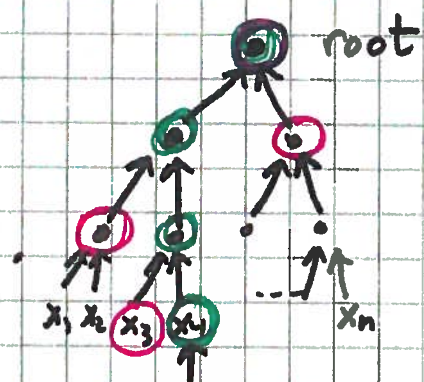
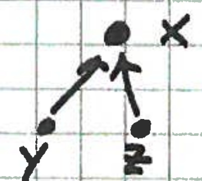
(5) **To authenticate a collection of n objects:**

Build a tree with n leaves $x_1, x_2, \ldots, x_n$ & compute authenticator for node as fn of values at children... This is a "Merkle tree":




Value at x

= h(value at y // value at z)

Root is authenticator for all n values $x_1, x_2, \ldots, x_n$

To authenticate $x_i$, give sibling of $x_i$ & sibling of all his ancestors up to root

Apply to: time-stamping data

authenticating whole file system

**Need:** CR

Used in bitcoin...

# Puzzles & Brute-Force Search

Want to create puzzle with solution known to creator that requires (on average) a fixed amount of work to solve.

Let $h: \{0,1\}^* \rightarrow \{0,1\}^d$ be a crypto hash fn
(e.g. SHA-256 with $d=256$)

The "puzzle" will be to invert $h$, i.e. solve $h(x)=y$ for $x$ given $y$.

restricted domain

To make this a puzzle, we restrict $x$ to be in a known set $S$ of possible solutions. Eg. $S=\{0,1\}^s$ for $s=40$.

To create a puzzle, pick $x \in S$ at random, compute $y=h(x)$.

Difficulty of solving $\approx |S|/2$ by brute-force search.

If $s \ll d$ there will be no "false solutions" — no collisions.

Can create multiple (keyed) puzzles $(k,y)$ means solving $h(k \| x)=y$ for $x \in S$.
Puzzle spec is $(h,k,s,y)$

Puzzle creator knows solution

restricted range

= Can also have puzzles where creator doesn't know solution with truncated hashes

$$h: \{0,1\}^* \rightarrow \{0,1\}^s$$

try $x$ at random until $h(x)=y$

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## Hash cash (Adam Back, 1997)

- Anti-spam measure
- Requires sender to provide "proof of work" ("stamp")
- Email without POW or from sender on whitelist is discarded.
- POW:

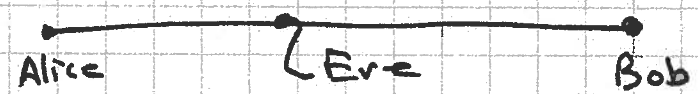
  solve puzzle $h(k, r)$ ends in 20 zeros

  where $k$ = sender ‖ receiver ‖ date ‖ time

  $r$ = variable to be solved for
- Include $r$ in header as POW
- easy for receiver to verify payment (POW)
- takes $\approx 2^{20}$ trials to solve
- doesn't work well against botnets ☹

## Merkle puzzles

- First "public key" system (really: key agreement)




Eve is passive eavesdropper.
How can Alice & Bob agree on a key?

Use puzzles (with restricted domain, so have unique solns)

$n = \#$ puzzles of difficulty $2^{s-1} = D$

(1) Bob choose $n$ values $x_1, x_2, \ldots, x_n$ from $S = \{0,1\}^s$

Bob computes $y_i = h(i \| x_i)$

Bob sends $(y_i, E_{x_i}(K_i))$ to Alice for $1 \le i \le n$, where $K_i \in_R \{0,1\}^{256}$

(2) Alice picks random $i$ from $[n] = \{1, 2, \ldots, n\}$
Alice solves $P_i$ for $x_i$
" decrypts to obtain $K_i$
" sends $h(K_i)$ to Bob

(3) Bob & Alice use $K_i$ to communicate secretly from then on.

Time for good guys $= \underbrace{\Theta(n)}_{\text{Bob}} + \underbrace{\Theta(D)}_{\text{Alice}}$

Time for Eve $= \Theta(n \cdot D)$

For $n = D = 10^9$, "almost practical"!

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## Hash function construction ("Merkle-Damgard" style)

- Choose output size $d$ (e.g. $d = 256$ bits)
- Choose "chaining variable" size $c$ (e.g. $c = 512$ bits)

  [Must have $c \geq d$; better if $c \geq 2 \cdot d$ ...]
- Choose "message block size" $b$ (e.g. $b = 512$ bits)
- Design "compression function" $f$

  $$f : \{0,1\}^c \times \{0,1\}^b \rightarrow \{0,1\}^c$$

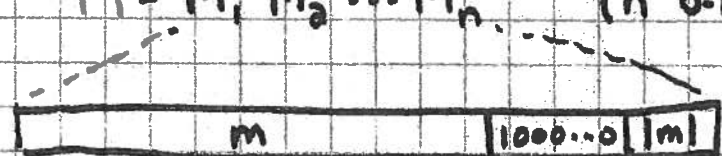
  [$f$ should be OW, CR, PR, NM, TCR, ...]
- Merkle-Damgard is essentially a "mode of operation" allowing for variable-length inputs:

  * Choose a $c$-bit initialization vector IV, $c_0$

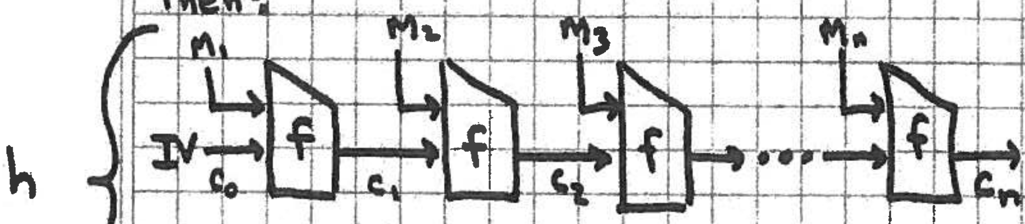
    [Note that $c_0$ is fixed & public.]
  * [Padding] Given message, append
    - $10^*$ bits
    - fixed-length representation of length of input

    so result is a multiple of $b$ bits in length:

    $$M = M_1 M_2 \cdots M_n \quad (n \text{ b-bit blocks})$$



Then:




$h(m) = c_n$ truncated to d bits

__Theorem:__ If f is CR, then so is h.

__Proof:__ Given collision for h, can find one for f by working backwards through chain. ∎

__Thm:__ Similarly for OW.

__Common design pattern for f:__

$$f(c_{i-1}, M_i) = c_{i-1} \oplus E(M_i, c_{i-1})$$

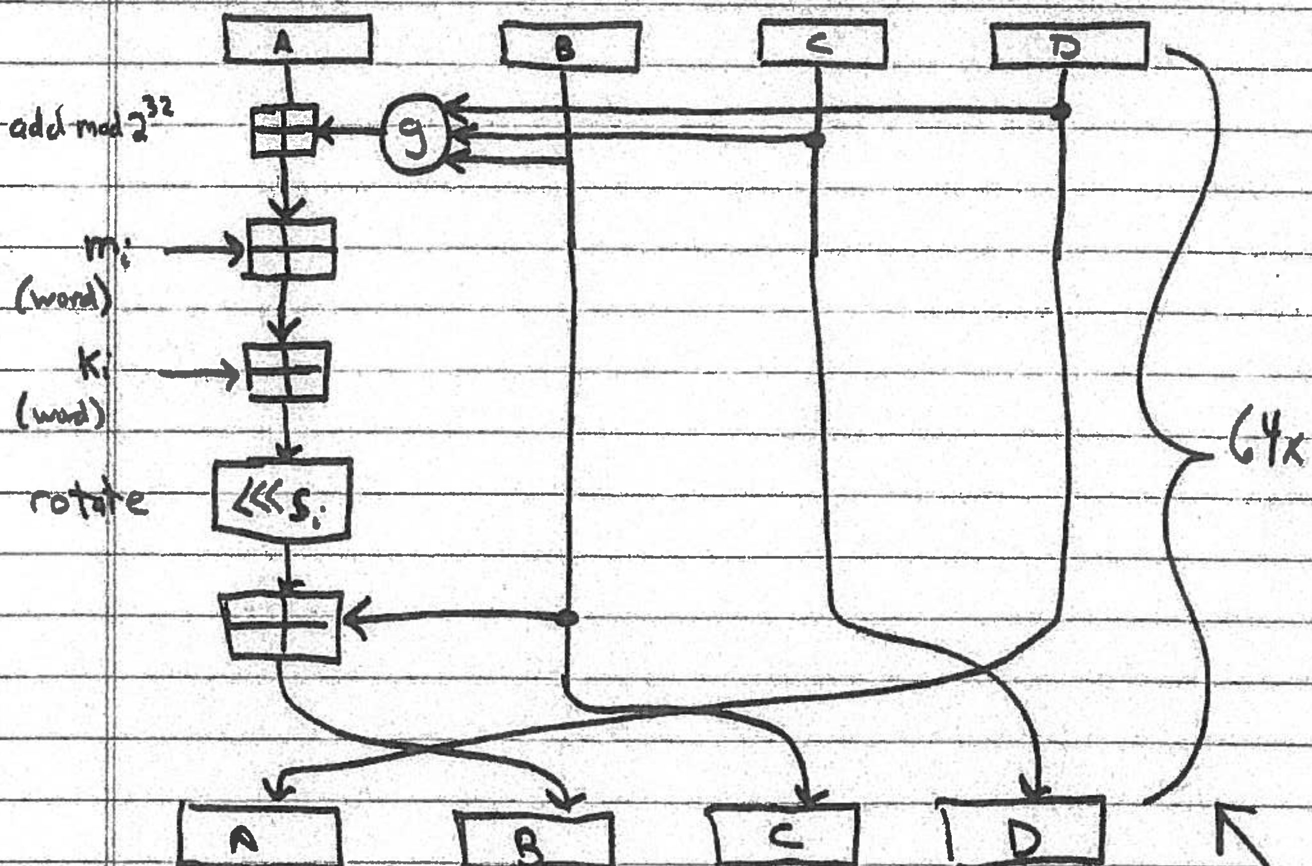
where $E(K,M)$ is an encryption function (block cipher) with b-bit key and c-bit input/output blocks.

(Davies-Meyer construction)

6.857 Rivest
[redacted]

## Typical compression function (MD5): (1991)

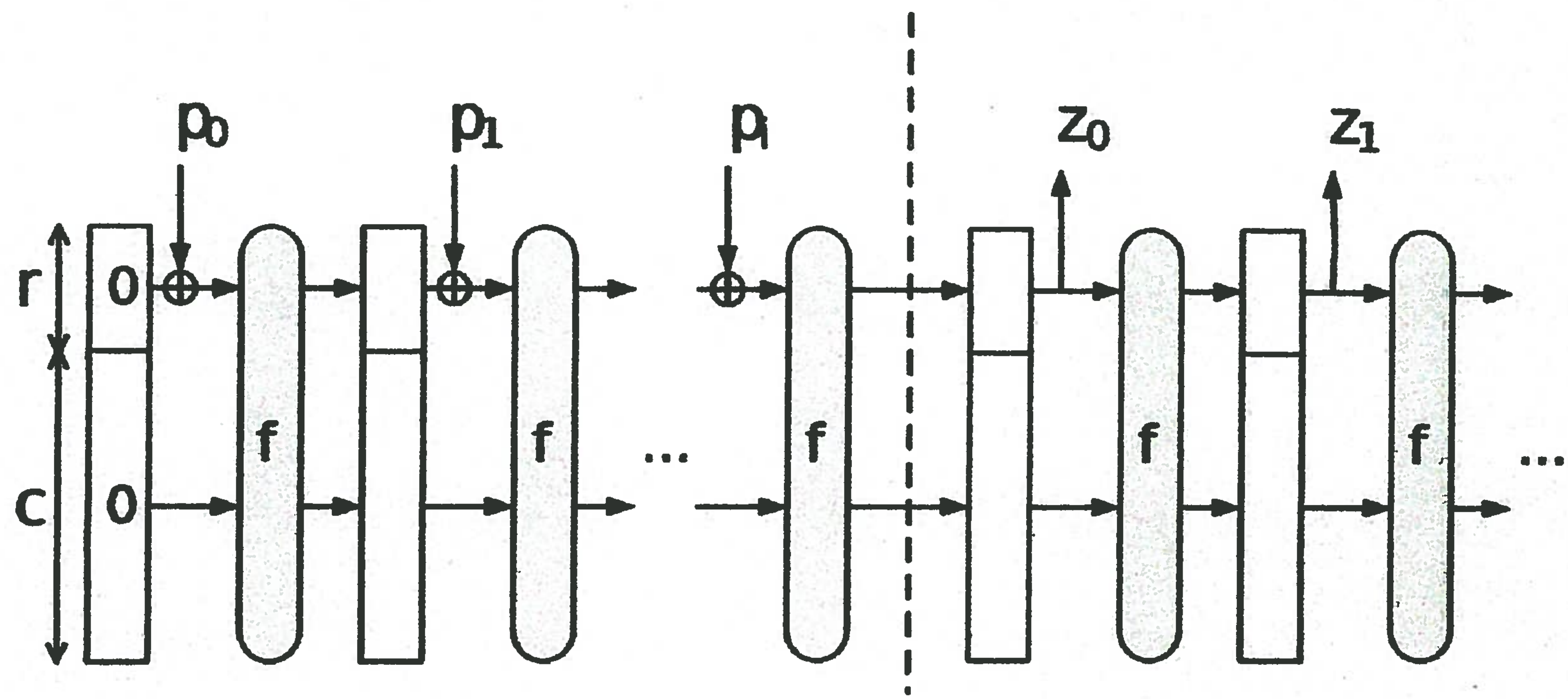
- chaining variable & output are 128 bits = 4 x 32
- IV = fixed value
- 64 rounds; each modifies state (in reversible way) based on selected message ~~block~~ word
- message block b = 512 bits considered as 16 32-bit words
- uses end-around XOR too around entire compression fn (as above)




Xiaoyun Wang discovered how to make collisions for MD4, MD5, ...
("Differential cryptanalysis")
[redacted]

$$g(x,y,z) = \begin{cases} xy \vee \bar{x}z \\ xz \vee y\bar{z} \\ x \oplus y \oplus z \\ y \oplus x\bar{z} \end{cases}$$ depending on round

Not CR! (Xiaoyun Wang 2005)




**Keccak Sponge Construction**

$d$ = output hash size in bits $\in \{224, 256, 384, 512\}$

$c = 2d$ bits

state size = $25w$ where $w$ = word size (e.g. $w=64$)

$c + r = 25w$

$r \geq d$ (so hash can be first $d$ bits of $z_0$)

Input padded with $10^*1$ until length is a multiple of $r$

$f$ has 24 rounds (for $w=64$), not quite identical (round constant)

$f$ is public, efficient, invertible function from $\{0,1\}^{25w}$ to $\{0,1\}^{25w}$

e.g. $d = 256$
$c = 512$
$r = 1088$
$w = 64$

Keccak = SHA-3