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Admin:

- Pset #3 & groups posted.
- Get your final project team & topic selected!

Today:

- Message Authentication Codes (MAC's)
  - HMAC
  - CBC-MAC
  - PRF-MAC
  - One-time MAC (problem statement)
- AEAD (Authenticated Encryption with Associated Data)
  - EAX mode (ref paper; pages 1-10 only)
  - Encrypt-then-MAC
- Finite fields & number theory

Readings:

Katz/Lindell Chapter 4

Paar/Pelzl Chapter 12

MAC (Message Authentication Code)

- Not confidentiality, but integrity (recall "CIA")
- Alice wants to send messages to Bob, such that Bob can verify that messages originated with Alice & arrive unmodified.
- Alice & Bob share a secret key K
- Orthogonal to confidentiality; typically do both (e.g. encrypt, then append MAC for integrity)
- Need additional methods (e.g. counters) to protect against replay attacks

$$\begin{array}{c} \text{Alice} \\ M \\ K \end{array} \xrightarrow{\quad M,\ MAC_K(M) \quad} \begin{array}{c} \text{Bob} \\ K \\ \end{array}$$

[Here M is message to be authenticated, which could be ciphertext resulting from encryption.]

- Alice computes $MAC_K(M)$ & appends it to M.
- Bob recomputes $MAC_K(M)$ & verifies it agrees which what is received. If $\neq$, reject message.

If MAC has t bits, then Adv has prob 2^{-t} of successful forgery.
Good MAC is (keyed) PRF.

Adversary (Eve) wants to forge $M', MAC_K(M')$ pair that Bob accepts, without Eve knowing K.

- She may hear a number of valid $(M, MAC_K(M))$ pairs first, possibly even with M's of her choice (chosen msg attacks).
- She wants to forge for M' for which she hasn't seen $(M', MAC_K(M'))$ valid pair.

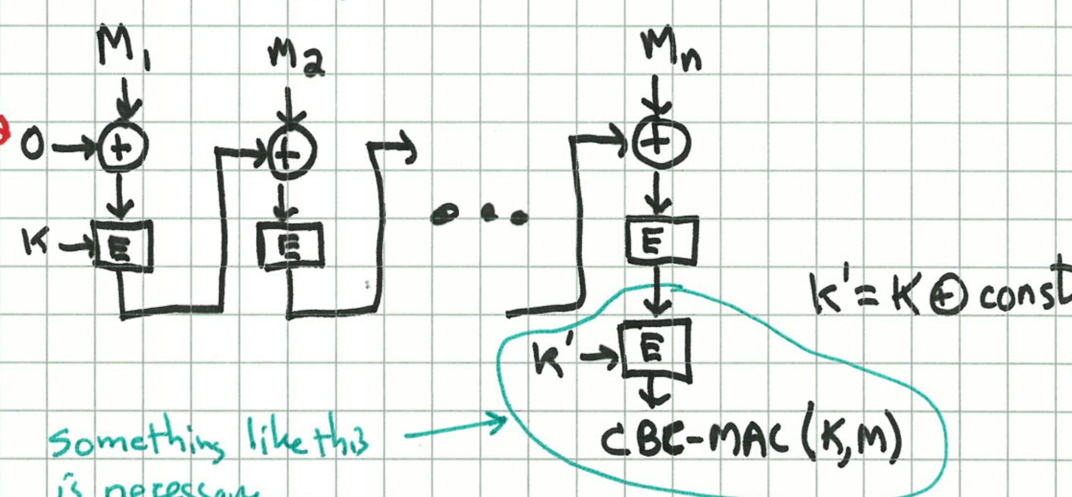
Two common methods:

$HMAC(K, M) = h(K_1 \| h(K_2 \| M))$

where $K_1 = K \oplus opad$, $K_2 = K \oplus ipad$ $\left\{ \begin{array}{l} opad, ipad \text{ are} \\ \text{fixed constants} \end{array} \right.$

$CBC\text{-}MAC(K, M) \cong$ last block of CBC enc. of M

note IV=0



Something like this is necessary...

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OK for h = RO
can be bad for h = iterative hash fn

MAC using random oracle (PRF):

$$MAC_K(M) = h(K \| M)$$

(OK if h is indistinguishable from RO, which means, as we saw, for sequential hash fns, that last block may need special treatment.)

One-Time MAC (problem stmt):

Can we achieve security against unbounded Eve, as we did for confidentiality with OTP, except here for integrity?

Here key K may be "use-once" [as it was for OTP].

$$\underset{K}{A} \xrightarrow{M,T} \underset{K}{B}$$

$T = MAC_K(M)$ ("tag")

- Eve can learn M,T then try to replace M,T with M',T' (where $M' \neq M$) that Bob accepts.
- Eve is computationally unbounded.

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	Confidentiality	Integrity
Unconditional	OTP ✓	One-time MAC ?
Conventional (symmetric key)	Block ciphers (AES) ✓	MAC (HMAC) ✓
Public-key (asymmetric)	PK enc.	Digital signature

Note: digital signature are unforgeable, but also have nonrepudiation, since only one copy of signing key exists.

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- **EAX mode**

[See pgs 1-10 of

The EAX Mode of Operation

by Bellare, Rogaway, & Wagner

]

Figure 3

- **Encrypt-then-MAC**

$C = Enc(K_1, M)$

$T = MAC(K_2, H \| C)$ ← C, not M!

↑ header: not encrypted, but authenticated

xmit: H, C, T

Two passes

Two keys

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Finite fields:

System $(S, +, \cdot)$ s.t.

- S is a finite set containing "0" & "1"
- $(S,+)$ is an abelian (commutative) group with identity 0

group laws
$$((a+b)+c) = (a+(b+c)) \quad \text{associative}$$
$$a+0 = 0+a = a \quad \text{identity } 0$$
$$(\forall a)(\exists b)\ a+b=0 \quad \text{(additive) inverses } b=-a$$

$$a+b = b+a \quad \text{commutative}$$

- $(S^*, \cdot)$ is an abelian group with identity 1

S^* = nonzero elements of S

group laws
$$(a\cdot b)\cdot c = a\cdot(b\cdot c) \quad \text{associative}$$
$$a\cdot 1 = 1\cdot a = a \quad \text{identity } 1$$
$$(\forall a \in S^*)(\exists b \in S^*)\ a\cdot b = 1 \quad \text{(multiplicative inverses) } b = a^{-1}$$

$$a\cdot b = b\cdot a \quad \text{commutative}$$

- Distributive laws: $a\cdot(b+c) = a\cdot b + a\cdot c$

$(b+c)\cdot a = b\cdot a + c\cdot a$ (follows)

Familiar fields: $\mathbb{R}$ (reals) are infinite
$\mathbb{C}$ (complex)

For crypto, we're usually interested in finite fields, such as $\mathbb{Z}_p$ (integers mod prime p)

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Over field, usual algorithms work (mostly).

E.g. solving linear eqns:

$$ax + b = 0 \pmod p$$

$$\Rightarrow \quad x = a^{-1} \cdot (-b) \pmod p \quad \text{is soln.}$$

$$3x + 5 = 6 \pmod 7$$

$$3x = 1 \pmod 7$$

$$x = 5 \pmod 7$$

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Notation: GF(q) is *the* finite field ("Galois field") with q elements

Theorem: GF(q) exists whenever
$$q = p^k, \; p \text{ prime}, \; k \geq 1$$

Two cases:

① GF(p) - work modulo prime p

Z_p = integers mod p = $\{0, 1, \ldots, p-1\}$

$Z_p^* = Z_p - \{0\} = \{1, 2, \ldots, p-1\}$

② GF(p^k) : $k > 1$

work with polynomials of degree $< k$
with coefficients from GF(p)
modulo fixed irreducible polynomial of degree k

Common case is GF(2^k)

Note: all operations can be performed efficiently

(inverses to be demonstrated)

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Construction of $GF(2^2) = GF(4)$

Has 4 elements.

Is not arithmetic mod 4. (where 2 has no mult. inverse)

elements are polynomials of ~~degree~~ degree <2 with coefficients mod 2 (i.e. in $GF(2)$):

	x	1
0	0	0
1	0	1
x	1	0
$x+1$	1	1

Addition is component-wise according to powers, as usual

$$(x) + (x+1) = (2x+1) = 1 \quad (\text{coefs. mod } 2)$$

Multiplication is modulo x^2+x+1
which is irreducible (doesn't factor)

	0	1	x	$x+1$
0	0	0	0	0
1	0	1	x	$x+1$
x	0	x	$x+1$	1
$x+1$	0	$x+1$	1	x

$x^2 \bmod (x^2+x+1)$ is $x+1$ (note that $x \equiv -x$ coefs mod 2)

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"Repeated squaring" to compute a^b in field

(Here b is a non-negative integer)

$$a^b = \begin{cases} 1 & \text{if } b=0 \\ (a^{b/2})^2 & \text{if } b>0,\ b \text{ even} \\ a \cdot a^{b-1} & \text{if } b \text{ odd} \end{cases}$$

Requires $\leq 2 \cdot \lg(b)$ multiplications in field (efficient)

$\approx$ a few milliseconds for $a^b \pmod p$ 1024-bit integers

$\approx$ $\Theta(k^3)$ time for k-bit inputs

Computing (multiplicative) inverses:

Theorem: (For GF(p) called "Fermat's Little Theorem")

In $GF(q)$ $(\forall a \in GF(q)^*)$ $a^{q-1} = 1$

Corollary: $(\forall a \in GF(q))$ $a^q = a$

Corollary: $(\forall a \in GF(q)^*)$ $a^{-1} = a^{q-2}$

Example: $3^{-1} \pmod 7$

$= 3^5 \pmod 7$

$= 5 \pmod 7$