

Admin:

Pset #1 due, & pset #2 out, on Monday 2/24.

Lectures by TA's next week (secret sharing & bitcoin)

Submit passwords (not real ones) to TA's. for pset #2. today

Project idea:

"Format-Transforming Encryption"

Shrimpton 2014 Real-World Crypto talk
slides in our "top-secret" folder
also see <https://fteproxy.org>

Today:

Crypto hash fns: applications & constructions

Applications: signatures
commitments
Merkle trees
Payword
Hash-cash

Construction: Merkle-Damgard
Sponge function

③ Digital signatures ("hash & sign")

PK_A = Alice's public key (for signature verification)

SK_A = Alice's secret key (for signing)

Signing: $\sigma = \text{sign}(SK_A, M)$ [Alice's sig on M]

Verify: $\text{Verify}(M, \sigma, PK_A) \in \{\text{True}, \text{False}\}$

Adversary wants to forge a signature that verifies.

- For large M, easier to sign $h(M)$:

$\sigma = \text{sign}(SK_A, h(M))$ ["hash & sign"]

Verifier recomputes $h(M)$ from M, then verifies σ .

In essence, $h(M)$ is a "proxy" for M.

- Need CR (Else Alice gets Bob to sign x, where $h(x) = h(x')$, then claims Bob really signed x' , not x.)
- Don't need OW (e.g. $h = \text{identity}$ is OK here.)

④ Commitments

- Alice has value x (e.g. auction bid)
- Alice computes $C(x)$ ("commitment to x ") & submits $C(x)$ as her "sealed bid"
- When bidding has closed, Alice should be able to "open" $C(x)$ to reveal x
- Binding property: Alice should not be able to open $C(x)$ in more than one way! (She is committed to just one x .)
- Secrecy (hiding): Auctioneer (or anyone else) seeing $C(x)$ should not learn anything about x .
- Non-malleability: Given $C(x)$, it shouldn't be possible to produce $C(x+1)$, say...

• How: $C(x) = h(r || x) \quad r \in_R \{0,1\}^{256}$

To open: reveal r & x

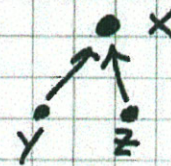
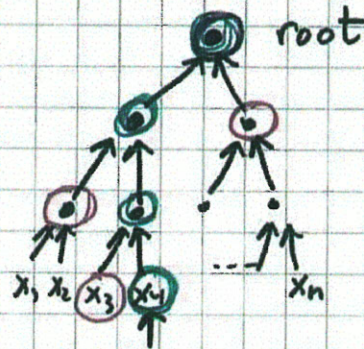
- Note that this method is randomized (as it must be for secrecy).

• Need: OW, CR, NM

(really need more, for secrecy, as $C(x)$ should not reveal partial information about x , even.)

⑤ To authenticate a collection of n objects:

Build a tree with n leaves x_1, x_2, \dots, x_n
 & compute authenticator for node as fn of values
 at children... This is a "Merkle tree":



value at x

$$= h(\text{value at } y \parallel \text{value at } z)$$

Root is authenticator for all n values x_1, x_2, \dots, x_n

To authenticate x_i , give sibling of x_i &
 sibling of all his ancestors up to root

Apply to: time-stamping data

authenticating whole file system

Need: CR

Hash-cash (by Adam Back)

- "Proof of work" by email sender
- Intent: reduce spam by making email "expensive" (computational)
- Sender must solve puzzle:

find r s.t.

$h(\text{sender, recipient, date, time, } r)$

ends in 20 zeros

- include r in header as "proof of work/payment"
- each for recipient to verify
- takes about 2^{20} trials to solve for r
- doesn't work against bot-nets 😞

Hash function construction ("Merkle-Damgard" style)

- Choose output size d (e.g. $d = 256$ bits)
- Choose "chaining variable" size c (e.g. $c = 512$ bits)
[Must have $c \geq d$; better if $c \geq 2 \cdot d \dots$]
- Choose "message block size" b (e.g. $b = 512$ bits)
- Design "compression function" f
$$f: \{0,1\}^c \times \{0,1\}^b \rightarrow \{0,1\}^c$$

[f should be OW, CR, PR, NM, TCR, ...]
- Merkle-Damgard is essentially a "mode of operation" allowing for variable-length inputs:

* Choose a c -bit initialization vector IV , c_0

[Note that c_0 is fixed & public.]

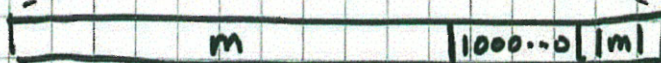
* [Padding] Given message, append

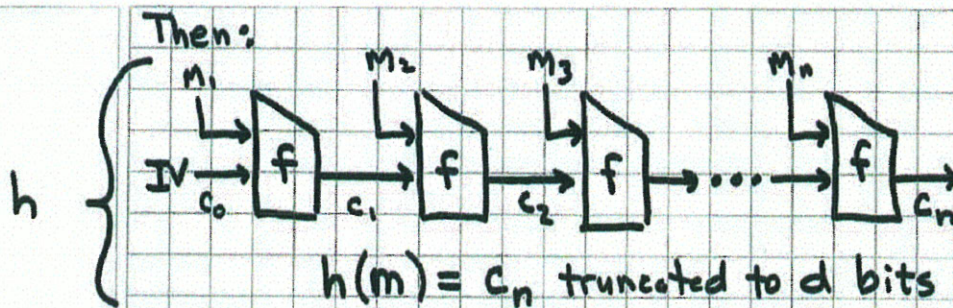
- 10^* bits

- fixed-length representation of length of input

so result is a multiple of b bits in length:

$M = M_1, M_2 \dots M_n$ (n b-bit blocks)





Theorem: IF f is CR, then so is h .

Proof: Given collision for h , can find one for f by working backwards through chain. ▣

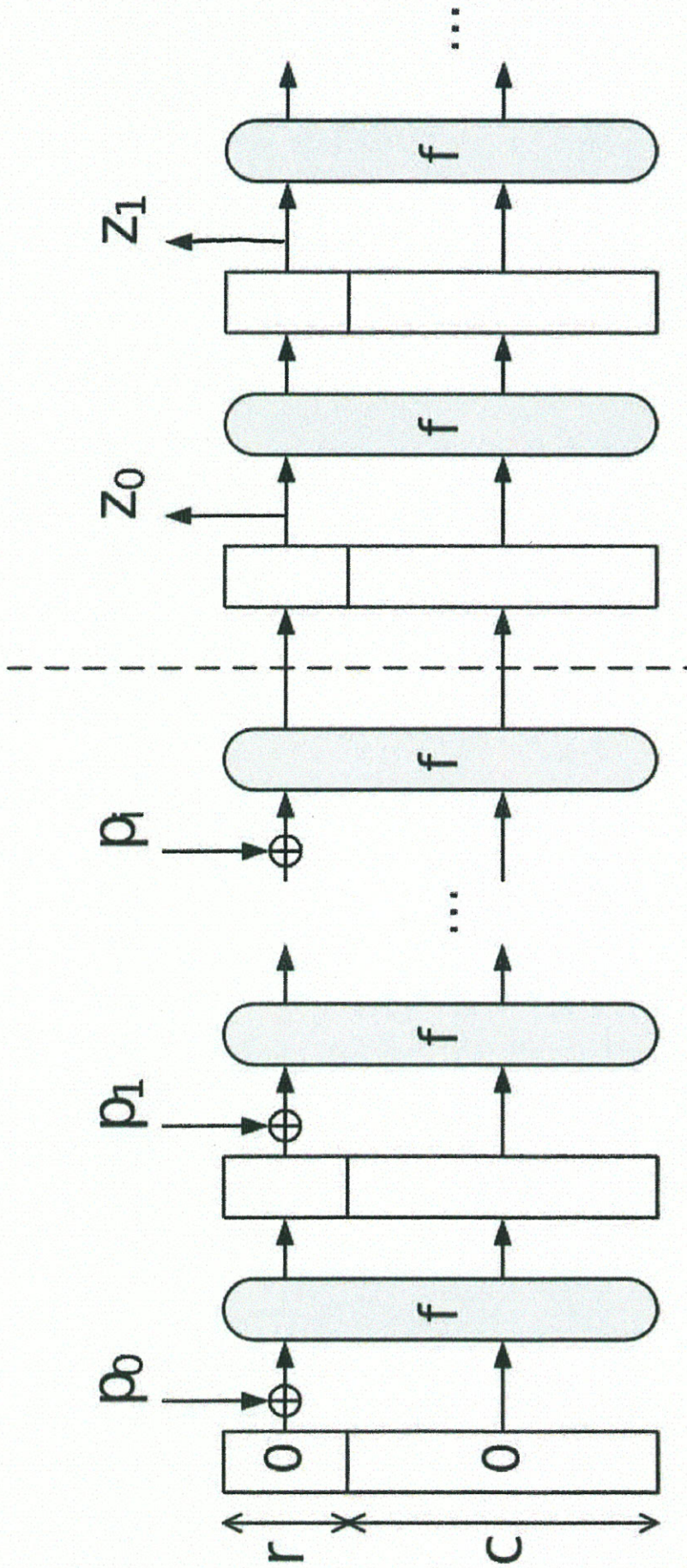
Thm: Similarly for OW.

Common design pattern for f :

$$f(c_{i-1}, M_i) = c_{i-1} \oplus E(M_i, c_{i-1})$$

where $E(K, M)$ is an encryption function (block cipher) with b -bit key and c -bit input/output blocks.

(Davies-Meyer construction)



Keccak Sponge Construction

$d = \text{output hash size in bits} \in \{224, 256, 384, 512\}$

$C = 2d$ bits

state size = $25w$ where $w = \text{word size (e.g. } w=64)$

$C+r = 25w$

$r \geq d$ (so hash can be first d bits of Z_0)

Input padded with 10^* until length is a multiple of r
 f has 24 rounds (for $w=64$), not quite identical (round constant)
 f is public, efficient, invertible function from $\{0,1\}^{25w}$ to $\{0,1\}^{25w}$

e.g. $d = 256$
 $c = 512$
 $r = 1088$
 $w = 64$

Keccak