TOPIC: 6.857 - Computer & Network Security

DATE: 4/10/13

FILE UNDER:

PAGE: L17.1

Admin:

Quiz 4/17 open notes

Discuss projects with TA's this week

Vinod talk Thu 4/11 32G-449 (Functional Enc.) 4pm

Vaikuntanathan

Today:

Digital Sigs. (cont.)

- Hash & sign
- PKCS
- PSS
- El Gamal dig sig
- Digital sig standard

PKCS, PSS } RSA

El Gamal dig sig, Digital sig standard } El-Gamal based

TOPIC:

DATE:

FILE UNDER:

Digital signatures

- Def of digital signature scheme
- Def of weak/strong existential unforgeability under adaptive chosen message attack.

$\Big\}$ see notes from last lecture

Hash & Sign:

For efficiency reasons, usually best to sign cryptographic hash $h(M)$ of message, rather than signing M. Modular exponentiations are slow compared to (say) SHA-256.

Hash function h should be collision-resistant.

TOPIC:

DATE:

FILE UNDER:

Signing with RSA — PKCS

- PKCS = "Public key cryptography standard"

  (early industry standard)

- Hash & sign method. Let H be C.R. hash fn.

- Given message M to sign:

  Let $m = H(M)$

  Define pad(m) =

  $$0x\ 00\ 01\ FF\ FF \ldots FF\ 00 \,\|\, \text{hash-name} \,\|\, m$$

  where # FF bytes enough to make $|pad(m)| = |n|$ in bytes.

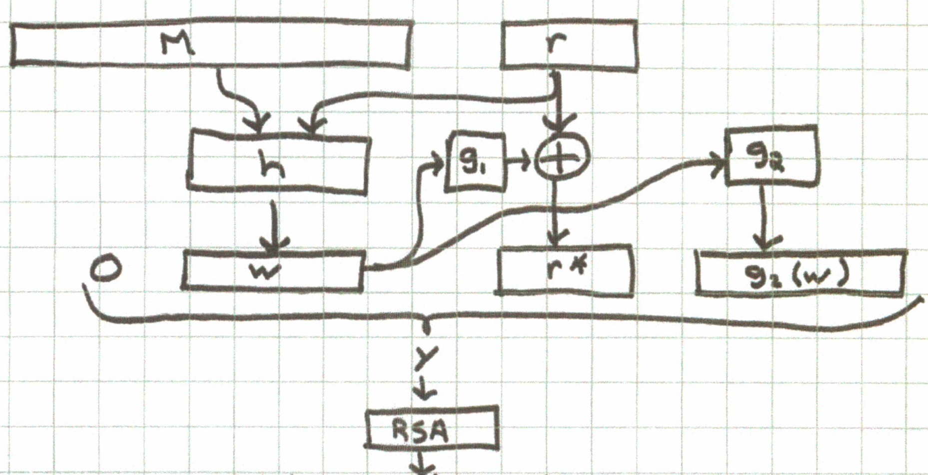
  where hash-name is given in ASN.1 syntax (ugly!)

- Seems secure, but no proofs (even assuming H is CR and RSA is hard to invert)

- $\sigma(M) = (pad(m))^d \pmod n$

PSS – Probabilistic Signature Scheme [Bellare & Rogaway 1996]

- RSA-based
- "Probabilistic" ≡ randomized [one M has *many* sigs]



$$\sigma(M) = y^d \pmod n$$

Sign(M): $r \xleftarrow{R} \{0,1\}^{k_0}$

$w \longleftarrow h(M \| r)$ $\qquad |w| = k_1$

$r^* \longleftarrow g_1(w) \oplus r$ $\qquad |r^*| = k_0$

$y \longleftarrow 0 \| w \| r^* \| g_2(w)$ $\qquad |y| = |n|$

output $\sigma(M) = y^d \pmod n$

Verify(M, σ): $y \leftarrow \sigma^e \pmod n$

Parse y as $b \| w \| r^* \| \gamma$

$r \leftarrow r^* \oplus g_1(w)$

return True iff $b = 0$ & $h(M \| r) = w$ & $g_2(w) = \gamma$

- We can model h, g_1, and g_2 as random oracles.

Theorem:

PSS is (weakly) existentially unforgeable against a chosen message attack in random oracle model if RSA is not invertible on random inputs.

El Gamal digital signatures

Public system parameters: prime p

generator g of Z_p^*

Keygen: $x \xleftarrow{R} \{0,1,\ldots,p-2\}$ $SK = x$

$y = g^x \pmod{p}$ $PK = y$

Sign(M):

$m = \text{hash}(M)$ CR hash fn into Z_{p-1}

$k \xleftarrow{R} Z_{p-1}^*$ [$\gcd(k, p-1) = 1$]

$r = g^k$ [hard work is indep of M]

$s = \dfrac{(m - rx)}{k} \pmod{p-1}$

$\sigma(M) = (r, s)$

Verify(M, y, (r, s)):

Check that $0 < r < p$ (else reject)

Check that $y^r r^s = g^m \pmod{p}$

where $m = \text{hash}(M)$

Correctness of El Gamal signatures:

$$\underbrace{y^r r^s = g^{rx} g^{sk} = g^{rx+sk} \stackrel{?}{=} g^m \pmod{p}}_{\equiv}$$

$$rx + ks \stackrel{?}{=} m \pmod{p-1}$$

or $$s \stackrel{?}{=} \frac{(m - rx)}{k} \pmod{p-1}$$

(assuming $k \in \mathbb{Z}^*_{p-1}$) $\blacksquare$

TOPIC:

DATE:

FILE UNDER:

Theorem: El Gamal signatures are existentially forgeable (without h, or h = identity (note: this is CR!))

Proof: Let $e \xleftarrow{R} Z_{p-1}$

$$r \leftarrow g^e \cdot y \pmod p$$

$$s \leftarrow -r \pmod p$$

Then (r,s) is valid El Gamal sig. for message $m = e \cdot s \pmod{p-1}$.

Check: $y^r r^s \stackrel{?}{=} g^m$

$$g^{xr}(g^e y)^{-r} = g^{-er} = g^{es} = g^m \checkmark \qquad \blacksquare$$

But: It is easy to fix.

Modified El Gamal (Pointcheval & Stern 1996)

Sign (M): $k \xleftarrow{R} Z_p^*$

$$r = g^k \pmod p$$

$$m = h(M \| r) \qquad \Leftarrow ***$$

$$s = (m - rx)/k \pmod{p-1}$$

$$\sigma(M) = (r,s)$$

Verify: Check $0 < r < p$ & $y^r r^s = g^m$ where $m = h(M \| r)$.

Theorem: Modified El Gamal is existentially unforgeable against adaptive chosen message attack, in ROM, assuming DLP is hard.

Digital Signature Standard (DSS - NIST 1991)

Public parameters (same for everyone):

q prime $\qquad |q| = 160$ bits

$p = nq+1$ prime $\qquad |p| = 1024$ bits

g_0 generates Z_p^*

$g = g_0^n$ generates subgroup G_q of Z_p^* of order q

Keygen:

$x \xleftarrow{R} Z_q$ $\qquad$ SK $\qquad |x| = 160$ bits

$y \leftarrow g^x \pmod{p}$ $\qquad$ PK $\qquad |y| = 1024$ bits

Sign (m):

Note: if k is reused for different messages m, one could solve for x so it is not secure. If k is reused for the same m, we obtain the same signature so this is not a problem. If k is different for the same m, it should be random and unknown (any known relation between the two k-s allows to solve for x)

Bottomline: All of the above are enforced by k chosen at random from Z_q^* for large enough q

$k \xleftarrow{R} Z_q^*$ $\qquad$ (i.e. $1 \le k < q$)

$r = (g^k \bmod p) \pmod{q}$ $\qquad |r| = 160$ bits

$m = h(M)$

$s = (m + rx)/k \pmod{q}$ $\qquad |s| = 160$ bits

<u>redo</u> if $r=0$ or $s=0$

$\sigma(M) = (r, s)$

Verify:

Check $0 < r < q$ & $0 < s < q$

Check $y^{r/s} g^{m/s} \pmod p \pmod q = r$

where $m = h(M)$

Correctness:

$$g^{(rx+m)/s} \stackrel{?}{=} r \pmod p \pmod q$$

$$\equiv \quad g^k = r \pmod p \pmod q \quad \checkmark$$

As it stands, existentially forgeable for h = identity.

Provably secure (as with Modified El Gamal)

if we replace $m = h(M)$ by $m = h(M \| r)$, as before.

Note: As with El Gamal, secrecy & uniqueness of k is essential to security.