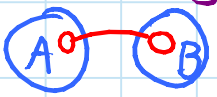
6.851	Class 11	Apr. 23, 2014

TODAY: Dynamic graphs
- new results

New results: (partial list)

Worst-case dynamic connectivity:
- recall best previous was $\Theta(\sqrt{n})$ update
- $O(\lg^4 n)$ insert, $O(\lg^5 n)$ delete,
  $O(\frac{\lg n}{\lg\lg n})$ query correct with high probability
  [Kapron, King, Mountjoy – SODA 2013]
  - idea: to find replacement edge 

    look for edge with odd # endpts. in A
    ⇒ in XOR of edge names ∀ endpt. ∈A, evenly occurring edges cancel (& finds edge if only one)
  - $\sum$ random ± 1 per edge ∀ endpt. ∈A estimates $\sqrt{\text{size of cut}}$
  - use sketching from streaming algorithms

Transitive closure: directed, $\exists s \to t$ path?

- $\geq m^{1/3-o(1)}$ update or query (& product $\geq m^{2/3-o(1)}$)
  or $\geq m^{4/3-o(1)}$ preprocessing
  ASSUMING no $O(n^{2-\varepsilon})$ algorithm for 3SUM
  given n integers, do any 3 sum to $\emptyset$?
  [Abboud & Williams – arXiv 2014]

Single-source reachability: directed, s fixed

- $O(m+n)$ update or query trivial
  & best known for sparse graphs
- $O(n^{1.495})$ query [Sankowski – SODA 2007]
  via fast Boolean matrix multiplication
- any $O(n^{2-\varepsilon})$ update & query, & $O(n^{3-\varepsilon})$ preproc.
  $\Rightarrow O(n^{3-\varepsilon})$ Boolean matrix multiplication [AW]
- incr/decremental: ditto for worst-case updates
- decremental: $O(n^{0.984+\varepsilon})$ expected am. update,
  $O(1)$ query [Henzinger, Krinninger, Nanongkai – STOC 2014]

Single-source reachability counting:

nodes reachable from fixed node s

- $\geq m^{1-o(1)}$ update or query, or $n^{w(1)}$ preproc. [AW]
  ASSUMING strong exponential time hypothesis
  $\forall \varepsilon > 0 \; \exists k$ s.t. k-CNF can't be solved in $2^{(1-\varepsilon)n} n^{O(1)}$
  (n = # variables)

Single-source shortest paths: directed & weighted

- $O(m+n\lg n) = O(n^2 \lg n)$ update/query via Dijkstra
- any incr/decremental $O(n^{2-\varepsilon})$ update & query & $O(n^{3-\varepsilon})$ preprocessing $\Rightarrow O(n^{3-\varepsilon})$ algorithm for all-pairs shortest paths with polynomial integer edge weights [AW]
- decremental $(1+\varepsilon)$-approximate: $O(n^{0.984+\varepsilon})$ [HKN]
- decremental $(1+\varepsilon)$-approx. unweighted undirected: $O(n^{0.8} \cdot \frac{n}{m} + m^{o(1)})$ update, $O(1)$ query [Henzinger, Krinninger, Nanongkai - SODA 2014]

Distance oracles: static weighted undirected graph, $O(1)$-time $(1+\varepsilon)$-approx. s-t shortest-path weight

- $O(\frac{1}{\varepsilon} n^{1+\varepsilon})$ space & $O(\frac{1}{\varepsilon} m n^{\varepsilon})$ preproc. [Thorup & Zwick - J.ACM 2005]
- $O(n \lg n \operatorname{polylg}\lg n)$ space & $O(1/\varepsilon)$ query for planar [Kawarabayashi, Sommer, Thorup - SODA 2013]

Problem 1: Add to link-cut trees the op.:

evert(x): reroot x's tree (represented) to have root x while preserving $O(\lg n)$ amortized/op.

[Sleator & Tarjan 1985]

$\Rightarrow$ $O(\lg n)$ dynamic connectivity in trees

Problem 2: Decremental single-source reachability in a DAG in $O(m+q)$ total time (q = #queries)

$\Rightarrow$ $O(1)$ amortized if all edges deleted

[Italiano – IPL 1988]