6.851	Class 8	Apr. 2, 2014

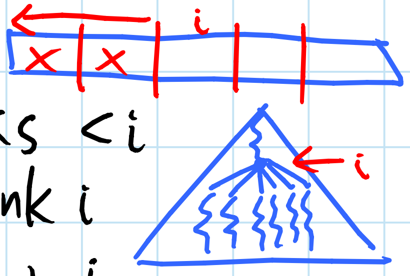
TODAY: More integers
- predecessor lower bound review
- signature/packed sorting review
  ≈ mergesort
- packed quicksort

Predecessor LB: $\min\{\log_a w, \log_b n\}$

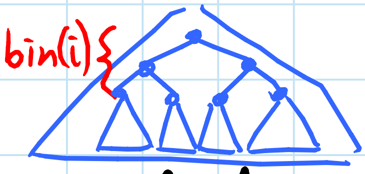
Alice's msg. = lg(space) $\qquad \overset{\|}{w}$ = Bob's msg.

- warning: lower bounds are hard!
- idea: construct tough instance (distribution)

① split words into k chunks, make data identical in chunks < i
⇒ predecessor determined by chunk i
  but query (Alice) doesn't know i
& make chunk i bad via ②, $w' = w/k$



② split n items into k′ chunks, make ith chunk items start bin(i)
⇒ which subproblem determined by i = lead
  lg k bits of query (not known to DS/Bob)
& make chunk i bad via ①, $n' = n/k'$
$w' = w - \lg k'$



Round elimination argument:

- consider t-round communication for ①
- Alice's first message has $\approx a/k$ useful bits

⇒ Bob can guess them with prob. $1/2^{a/k}$

⇒ error prob. $1 - 1/2^{a/k} \approx a/k$ ... actually $\sqrt{a/k}$

- left with Bob-first comm. for ②
- eliminate Bob's message, error prob: $+\sqrt{b/k'}$
- left with Alice-first comm. for ①
- after t such round eliminations, left with 0-message protocol for ①

⇒ error prob. must be $\geq 1/2$ if $n' \geq 2$ & $w' \geq 1$ (nontrivial instance)

i.e. $t \lesssim \min\{\log_k w, \log_{k'} n\}$

- set $k = at^2$ & $k' = bt^2$

⇒ if $t \lesssim \min\{\log_{\underbrace{at^2}_{=O(a^3)}} w, \log_{\underbrace{bt^2}_{=O(b^3)}} n\}$

$t = O(\lg n)$ & $a \geq \lg n$ ↷ $\quad$ ↶ $t = O(\lg w) = O(\lg b)$

$$= \Theta(\min\{\log_a w, \log_b n\})$$

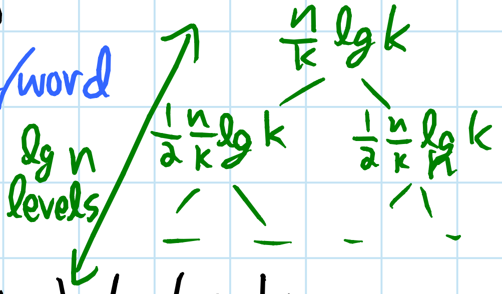
then
$$\begin{aligned} \text{error} &= t\left(\sqrt{a/k} + \sqrt{b/k'}\right) \\ &= t\left(1/t + 1/t\right) \\ &= 1/3 \quad \text{with appropriate constants} \\ &\ngeq 1/2 \quad \text{CONTRADICTION.} \end{aligned}$$

☐

Packed sorting: n b-bit ints. with $w = \Omega(b \overbrace{\lg n \lg\lg n}^{k})$

= mergesort with ints. packed in n/k words

$$\Rightarrow T(n) = 2T(n/2) + O(\underbrace{\lceil \tfrac{n}{k} \rceil}_{\text{\# words}} \underbrace{\lg k}_{\text{merge/word}})$$



$$= O(\tfrac{n}{k} \lg k \cdot \lg n)$$
$$= O(n)$$

- merge via bitonic sorting + bit tricks

Signature sort: $O(n)$ time for $w \geq \lg^{2+\varepsilon} n \cdot \lg\lg n$

- break integers into $\lg^{\varepsilon} n$ chunks of $\lg^2 n \cdot \lg\lg n$
- hash each chunk to $\lg n$ bits

$\Rightarrow \lg^{1+\varepsilon} n$-bit signature for each integer

- sort them via packed sorting
- fix chunk order of each node in trie by recursively sorting (node, chunk, edge index)
  - → get correct permutation on edges
- $b' = b/\lg^{\varepsilon} n + O(\lg n)$

$\Rightarrow$ after $1/\varepsilon$ recursions, $b' = O(w/\lg^{1+\varepsilon} n)$

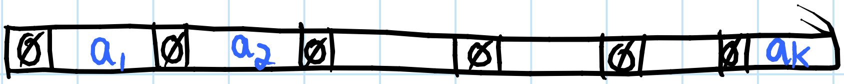
$$= O(w/\lg n \lg\lg n)$$

$\Rightarrow$ can use packed sorting to finish

Problem: Packed quicksort $w = \Omega(b \overbrace{\lg n \lg\lg n}^{k})$

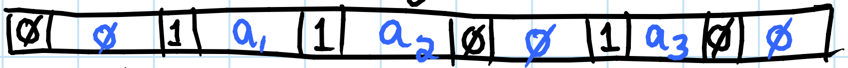
= quick sort with ints. packed in n/k words
↳ choose partition element x (e.g. random)
↳ partition array into $\le x$ & $>x$
↳ recursively sort
↳ concatenate

① partition 1 word with k elements:



into 2 words storing elts. $\le x$ & $>x$ in $O(1)$ time on word RAM

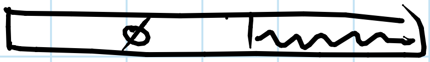
* ② given 2 words with $j_1, j_2 \le k/4$ elts. spread out:



combine into 1 word with all $j_1 + j_2$ elts. in $O(\lg k)$ expected time on word RAM

? ②' given 1 word with $j \le k$ elements spread out:

compactify to right so that $\ge$ constant fraction density



in polylg k time?

③ partition $O(n/k)$ words each with $\Theta(k)$ elts. into $O(\lceil n_{\le x}/k \rceil)$ words each with $O(k)$ elts. $\le x$ & $O(\lceil n_{>x}/k \rceil)$ words each with $O(k)$ elts. $>x$ in $O(\frac{n}{k} \lg k)$ expected time on word RAM