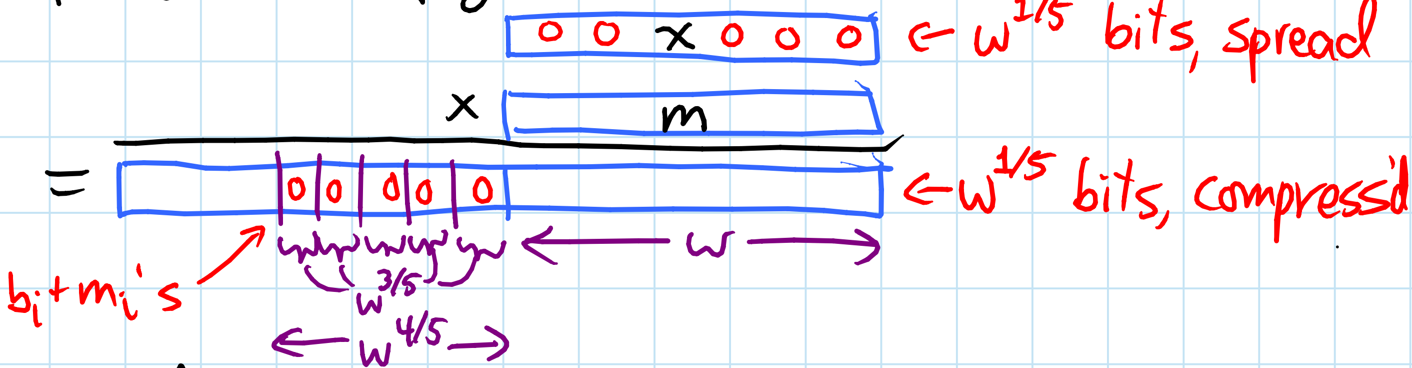
6.851	Class 7	Mar. 19, 2014

Approximate sketch: review

- given w-bit x & $w^{1/5}$ bits $b_0 < b_1 < \cdots < b_{r-1}$
- mask out those bits: x AND $\sum 2^{b_i} \to x'$
- want to compress x' to fit in $O(w^{4/5})$ bits
- plan: multiply, mask, and shift:



- challenge: must avoid collisions/carries in $x' \cdot m = \sum_i \sum_j x_{b_i} \cdot 2^{b_i + m_j}$ (if $m = \sum_j 2^{m_j}$)
- set $m_i = w^{3/5}\left(i + \left\lfloor \frac{w - b_i}{w^{3/5}} \right\rfloor\right) + \underbrace{m_i'}_{< w^{3/5}}$ right?

  $\Rightarrow m_i + b_i$ lies in $[w + i \cdot w^{3/5}, \; w + (i+1) \cdot w^{3/5})$

  $\Rightarrow$ linearly ordered
- set m_i' to avoid $\underbrace{m_x'}_{i} + \underbrace{b_y}_{w^{1/5}} - \underbrace{b_z}_{w^{1/5}}$ modulo $w^{3/5}$

  counting: $\Rightarrow < w^{3/5}$ bad

  $\Rightarrow m_i' + b_y \neq m_x' + b_z \mod w^{3/5}$ for $i \neq x$

  $\Rightarrow m_i + b_y \neq m_x + b_z$ (unless $(i, y) = (x, z)$)

  for $i = x \Rightarrow y \neq z$, also $\cancel{m_i} + b_y \neq \cancel{m_y} + b_z$ (though can be $\equiv$ mod $w^{3/5}$)

Problem: Van Emde Boas DS with $\sqrt{\ }$ clusters storing hash table of nonempty clusters

- what's wrong with "proof" of $O(n)$ space?
- how much space really?

(thanks to Vladimír Čunát)

- fix 1: indirection
- fix 2: $w' = w/2$, so store subwords in smaller vEB's

$\Rightarrow O(n)$ words of space