6.851	Class 4	Feb. 26. 2014

TODAY: Dynamic Optimality
- geometry → BSTs
  - online
  - offline

Summary:

Lower bounds:

$\Theta(\max \text{IRB})$

Signed Greedy

$\mathbin{/\!/}$ $\quad$ $\geq$

Wilber 1 (P) $\quad$ Wilber 2

independent rectangle bounds

Upper bounds:
- Splay
- Greedy
- Tango

Splay, Greedy: conjectured $O(1)$-competitive

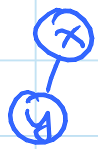
Tango — proved $O(\lg \lg n)$-competitive

Geometry → offline BST:

- treap: there's a unique tree that's a BST w.r.t. key & a heap w.r.t. priority
  - root = min priority item
  - BST property splits remainder
  - recurse on left & right subtrees
- here: key = key, priority = next touch time
- priorities aren't unique (>1 touch/time) so neither is treap

→ like B-tree

- multitreap: there's a unique tree that's search tree w.r.t. key & strict heap w.r.t. priority

 $\Rightarrow x < y$ & $\boxed{x \; y} \Rightarrow x = y$

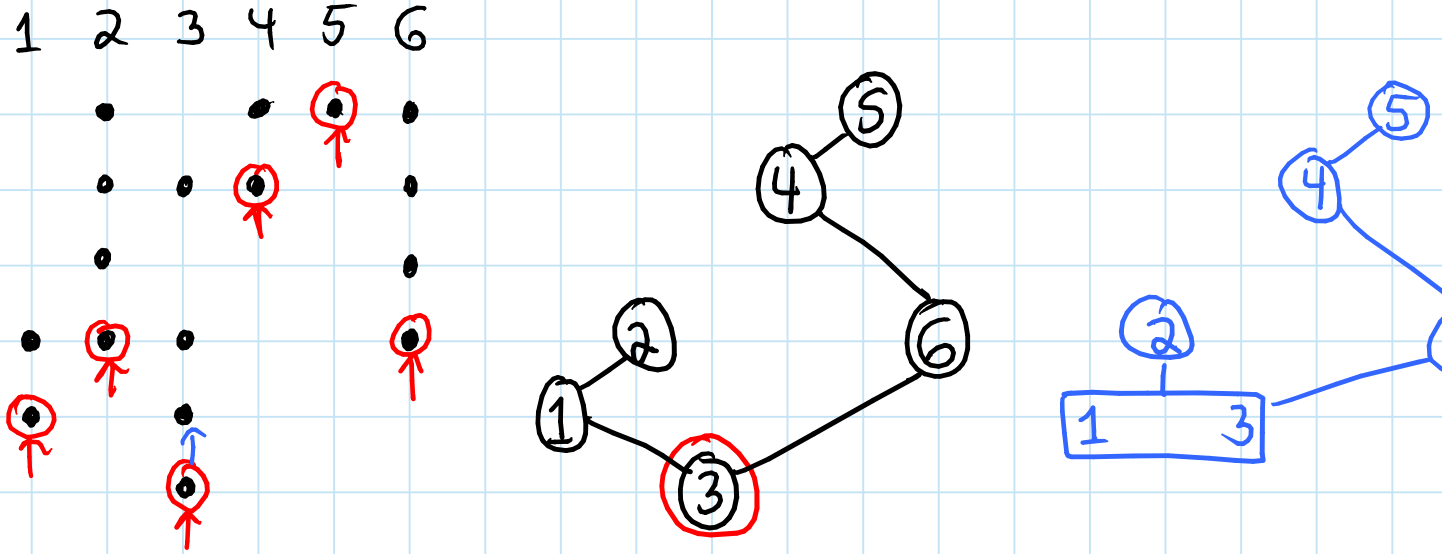
(strict increase) (only equal priorities in a single node)

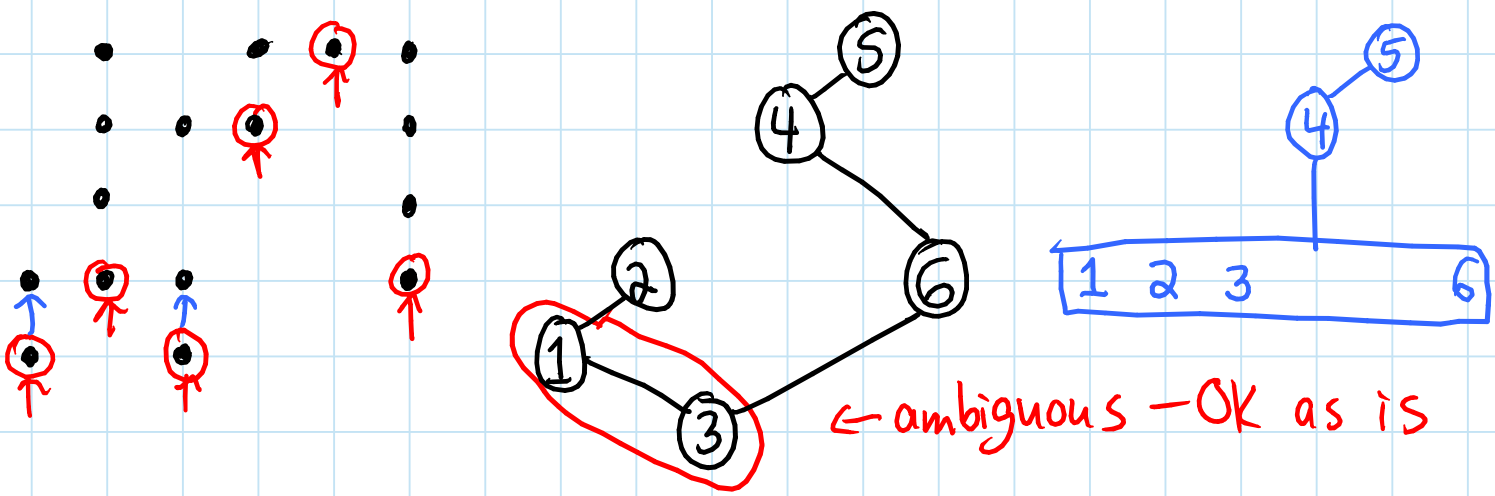
  - root consists of all min priority items
  - search tree property splits remainder
  - recurse

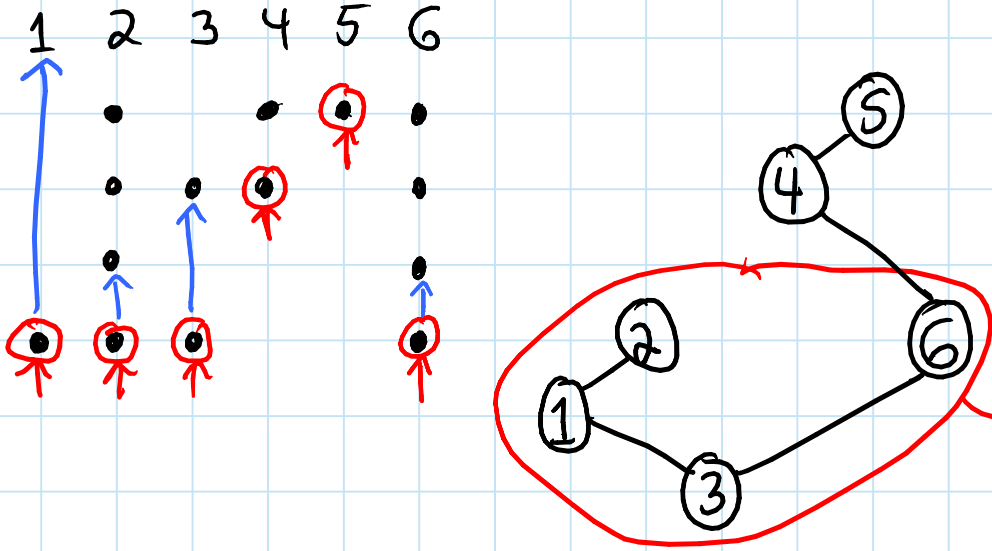
- effectively we maintain the unique multitreap
- really maintain a treap that "disambiguates" the multitreap

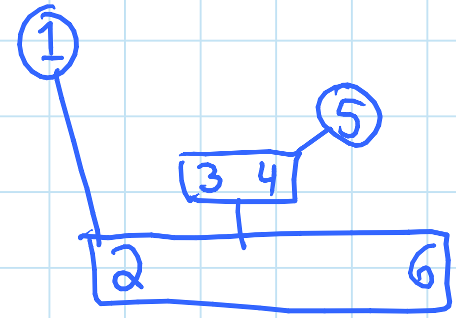
Example:

next touch time touch next multitreap

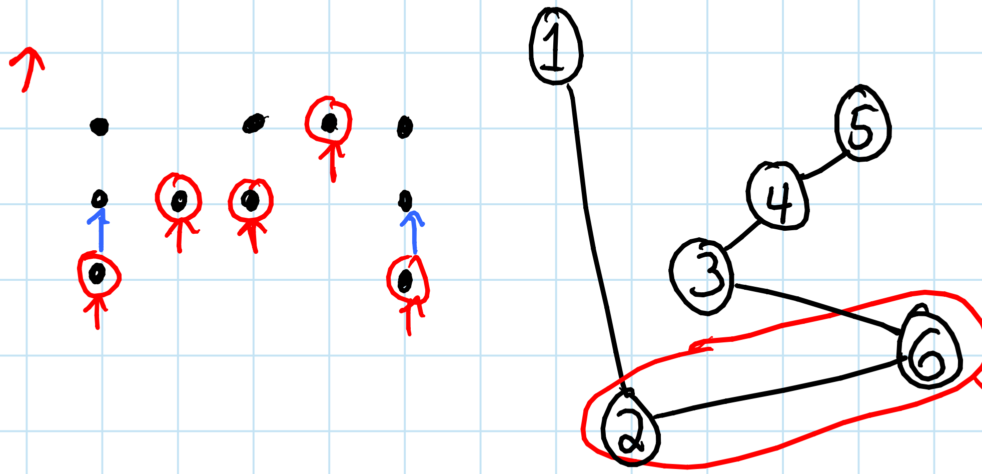


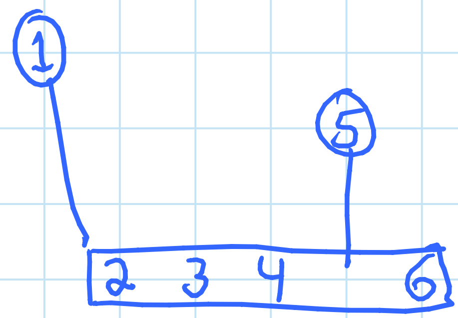




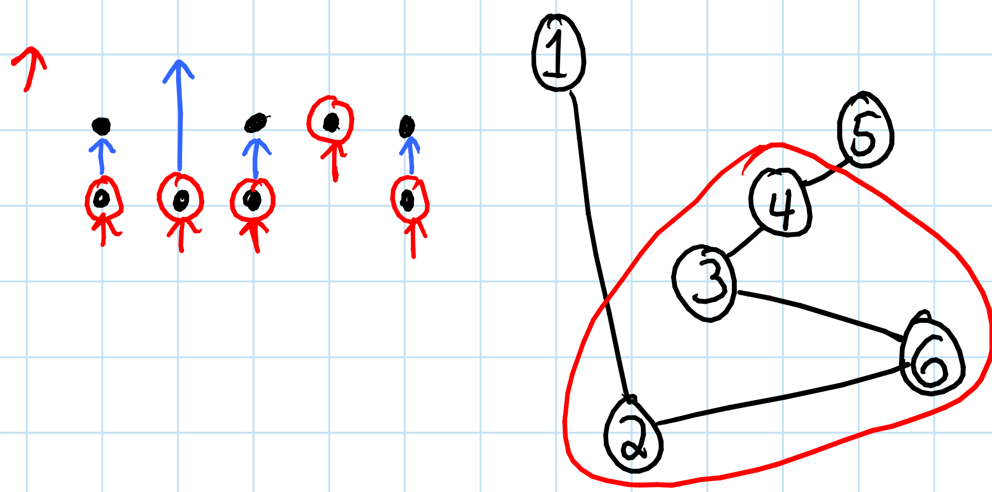


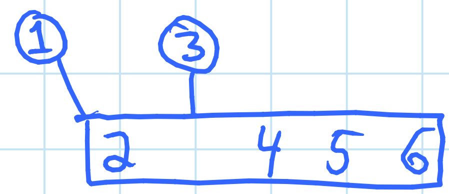
2,6 to root



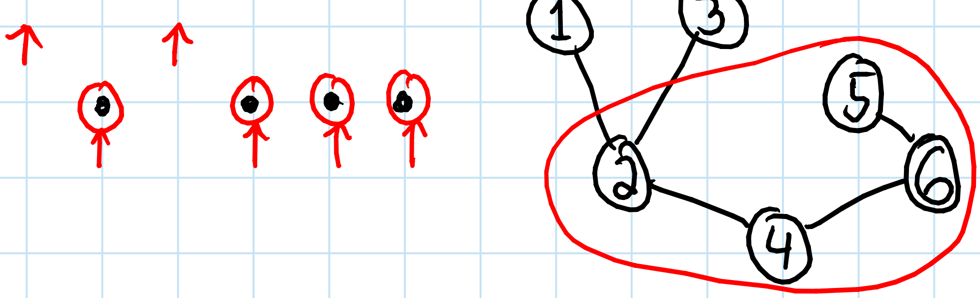


OK as is





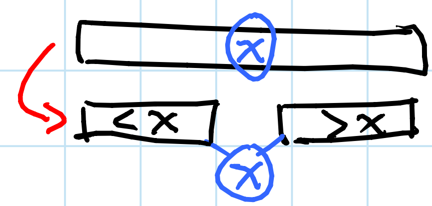
2,4,6 to root
(can't move 5
but it goes to
root too)



done

Geometry → online BST:

- maintain multitreap like right column
  but with more ambiguity
- initially 1 big ambiguous node
- when touching a node x:
  split ambiguous node
  into $<x$, x, $>x$



- to re-organize subtree of touched nodes:
  merge into one ambiguous root node
  (note all just touched ⇒ not ambiguous)
- same cost if we have:

Split trees: (ambiguous nodes)

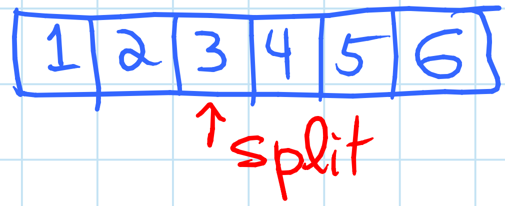
- make-tree$(x_1, x_2, \dots, x_k)$ — costs $\Theta(k)$
- $T.\text{split}(x) \to (T_{<x}, x, T_{>x})$ — costs $O(1)$ amortized

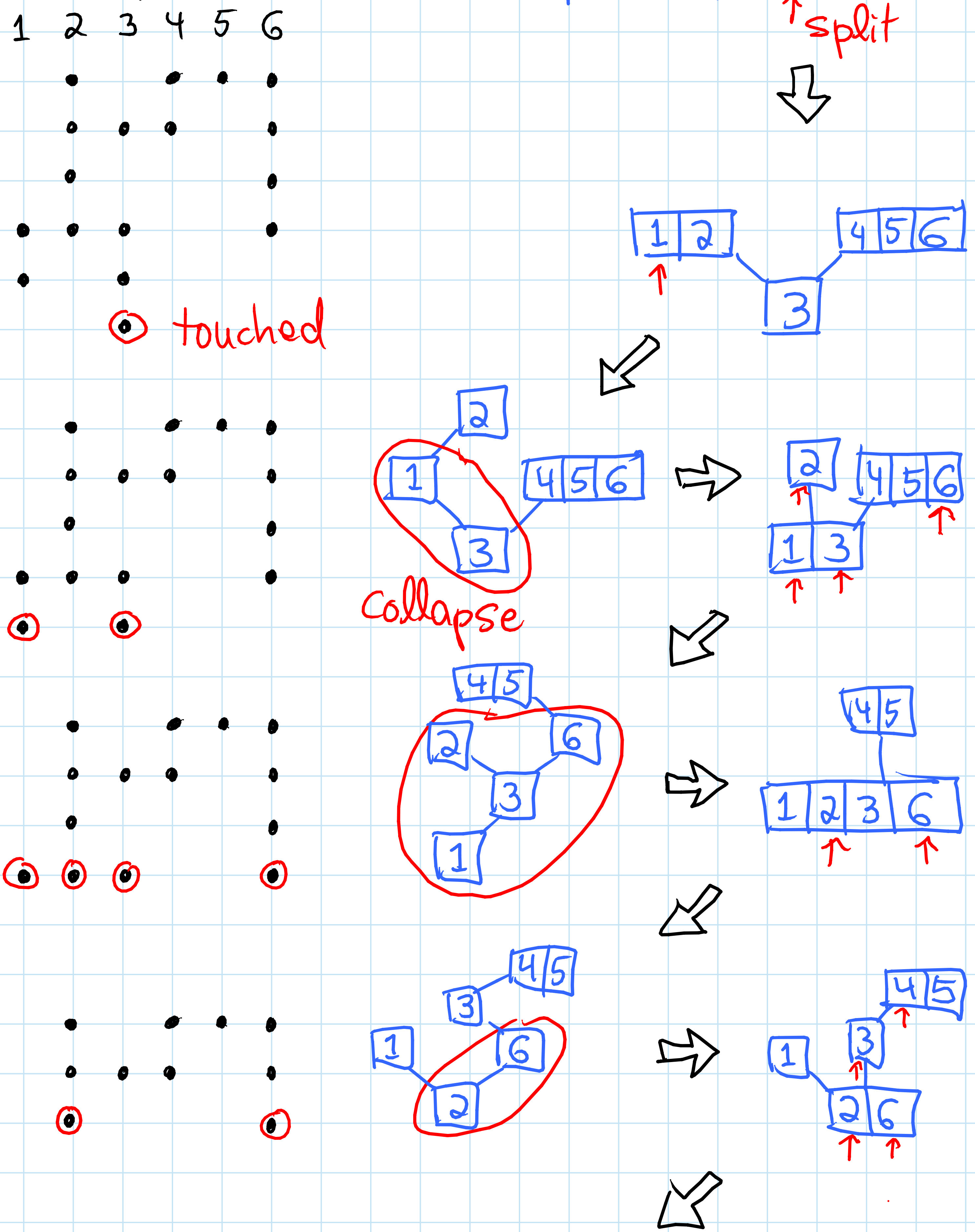
Details:

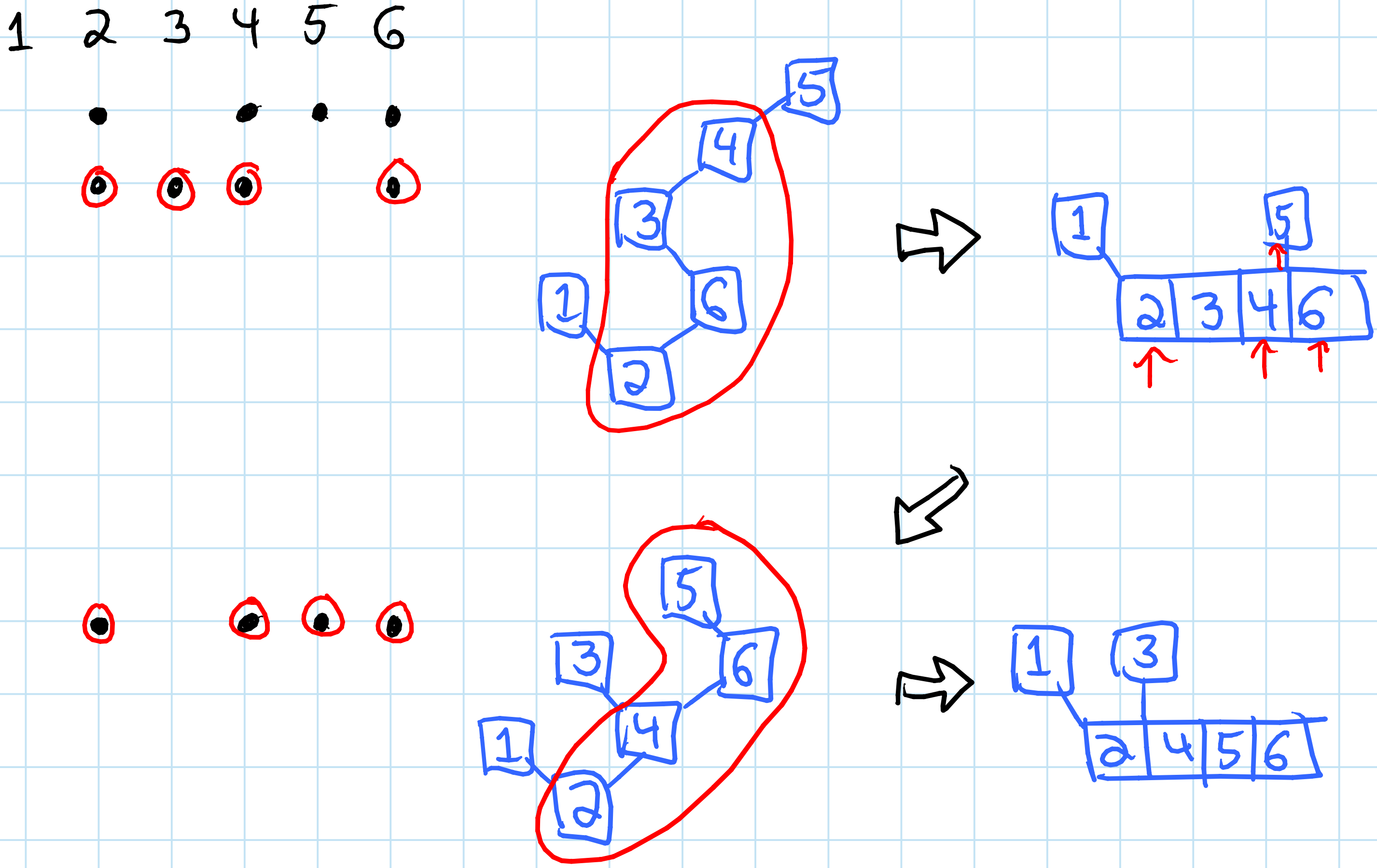
- $O(\min\{\lg|T_{<x}|, \lg|T_{>x}|\})$ split suffices
  via potential $\Phi = \sum_{\text{split tree } T} (|T| - \lg|T|)$

- B-trees with min & max augmentation
- simulate by BST (interleaving min/max)
- combine multiple split trees into BST
  similar to Tango trees (marked nodes)

Example: initial multitreap:






1 2 3 4 5 6
1 2 3 4 5 6
1 2 3 4 6 5
1 3 2 4 5 6
1 3 2 4 5 6

Problem 1: Show that assuming accesses X are all to distinct keys (x coords.) affects $|OPT(X)|$ by at most $O(|X|)$.
i.e. transform $X \to X'$ by spreading out identical keys (but otherwise preserving order) such that $|OPT(X')| \le |OPT(X)| + O(|X|)$
(3 suffices)

Problem 2: Prove a logarithmic separation between the BST & pointer-machine models (for successful searches)
i.e. construct an infinite family of access sequences $x_1, x_2, \ldots, x_m \in \{1, 2, \ldots, n\}$ such that $OPT_{BST}(X) = \Theta(m \lg n)$
& $OPT_{PM}(X) = \Theta(m)$

Discuss relationship between open problems, projects, papers, and authorship