6.851 Advanced Data Structures (Spring'10)

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$\boxed{\text{Problem 5}}$ Sample Solutions

Cartesian trees in linear time.

We will add the elements according to their order in A. If the current element x is greater than the previously added one, we just add it as its right child. Otherwise, we walk up the three until we find an element that is smaller then the current one. We then attach x as its right child, and attach it's previous right child as x's left child.

It is clear that with every step we make towards the root we reduce the path from the last element to the root by one. Having total of n elements, the maximum number of steps we make towards the root is also n. Therefore, the amortized time of insertion is $O(1)$.

Space requirements for integer data structures.

1.

Solving the reccurence on the size of a vEB tree for the universe u we get:

$$C(u) = (\sqrt{u} + 1)C(\sqrt{u}) + O(1)$$

We get $C(u) = O(u)$

2.

Conside the worst case, in which the prefixes diverge as early as possible. In each level i there are 2^i different prefixes. We reach n different prefixes at level $\log n$, at which each table can take up to $O(n)$ space.

Therefore our new bound is $O(n(\log \frac{u}{n} - 1))$