6.851 Advanced Data Structures (Spring'10)

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Problem 1 Sample Solutions

Transposing a matrix. Consider a point set $\{(x_i, i)\}$ of k^2 points on a $k^2 \times k^2$ lattice representing the access sequence. For each point (x_i, i) we introduce three new points at $(x_i - 1, i)$, $\left(k \left\lfloor \frac{x_i}{k} \right\rfloor, i\right)$ and $\left(k \left\lceil \frac{x_i}{k} \right\rceil, i\right)$.

The newly formed set is *Aborally Satisfied*, hence it represents a valid BST execution. The set contains $O(k^2)$, giving amortized cost of $O(1)$ per access.

Logarithmic redux. Consider the access sequence, the point set $X = \{(x_i, i)\}$ of m points on a $n \times m$ lattice. Let $\hat{x}$ be the median of all $x \in X$. Inserting m points $(\hat{x}, i)$ will ensure that each rectangle connecting a point left of or at $\hat{x}$ and a point right of or at $\hat{x}$ contains a point.

Now consider the two subsets of X, $X_{x \leq \hat{x}}$ and $X_{x \geq \hat{x}}$, each with at most m points, and at most $\frac{n}{2}$ distinct x values.

We recursively apply the same technique, to obtained point set that is *Aborally Satisfied.* We get the number of newly inserted points by solving the recursion $N(m, \frac{n}{2}) = 2N(m, \frac{n}{2}, n) + m$.

The total number of accesses is then $O(N(m, n) + m) = O(m \log n + m) = O(m \log n)$.