6.851 Advanced Data Structures (Spring'10)

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Problem 8 *Due: Thursday, Apr. 8*

Be sure to read the instructions on the assignments section of the class web page.

Cuckoo Hashing. We pick two hash-functions $f, g\colon [u] \to [m]$ uniformly at random. Let S be the set of keys we want to store by cuckoo hashing. We define the *cuckoo graph* as done in the lecture: its nodes are the cells of the table, and we have an edge $(f(x), g(x))$, for all $x \in S$. Assume further, that the size of the table is $m = 6|S|$.
Show that with probability at least $1/2$ the cuckoo graph contains no cycle.
Hint: Use the analysis by counting (similar to the "2-cycle case" in the cuckoo hashing analysis).

Conditional Expetations. Let G be a simple graph with vertex set V and edge set E. A cut of a set of vertices $V' \subseteq V$ is the number of edges that have one endpoint in V' and the other in $V \setminus V'$. The NP-complete `MaxCut` problem asks for the largest cut.
A simple randomized approximation problem works as follows: Throw for every vertex a coin. If we got "tails" we add it to V' otherwise not. In the end an edge is with probability $1/2$ in the cut, so the expected value of the cut for V' is $|E|/2$. Since every cut is at most $|E|$ we have a 2-approximation.
Use the concept of conditional expectations to de-randomize this algorithm.