

6.851 ADVANCED DATA STRUCTURES (SPRING'07)

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Problem 8 – Solution

Range Queries in 2D. For each $y \in \{1, \dots, n\}$, let $A[y] = \min\{x \mid (x, y) \in S\}$. When we want to query the range $[0, b] \times [c, d]$, we use an RMQ query to find $t = \min\{A[y] \mid y \in [c, d]\}$. If $t > b$, the range is empty; otherwise, it contains at least one point.

From Segment Stabbing to Existential dominance Queries in 2D. Replace a segment $[a, b]$ by the point $(a, u - b)$ in two dimensions. Now, for some x , we query the range $[0, x] \times [0, u - x]$. We have $(a, u - b) \in [0, x] \times [0, u - x] \iff a \leq x, u - b \leq u - x \iff a \leq x \leq b \iff x \in [a, b]$.

From Colored Predecessor to Segment Stabbing If we find two consecutive points of the same color, we can eliminate the second one, and the result of any predecessor query will not change. Thus, we can assume colors alternate in the sorted list of points. By symmetry, assume the first point is red. For every i , create a segment between the $2i$ -th and $(2i + 1)$ -st points (observe that the segments are disjoint). Now, if x stabs a segment, its predecessor has an even index, so it is blue. Otherwise, the predecessor is red.