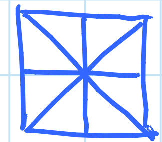
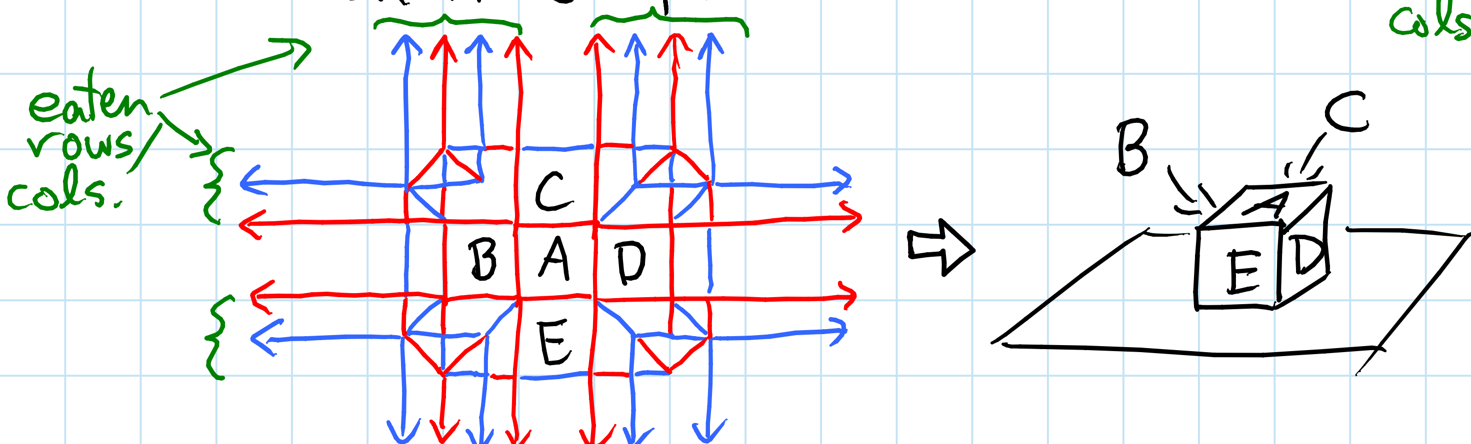
6.849	Lecture 5	Sept. 22, 2010

Universal hinge patterns: (for origami transformers)
[Benbernou, Demaine, Demaine, Ovadya 2010]

- suppose crease pattern required to be subset of fixed "hinge pattern"
  (e.g. Origamizer uses completely different creases for every model)
- $n \times n$ box-pleat pattern can make any polycube of $O(n)$ cubes, seamless: 
  - cube gadget turns $O(1)$ rows & columns into a cube sticking out of sheet ~ even if bumps elsewhere (not in eaten rows/cols.)



- to make a tree of cubes: (= any polycube)
  - make a leaf
  - conceptually remove it
  - repeat

  } "postorder traversal"
- actually need to reserve space ahead of time for all the cube gadgets

- $\Theta(n)$ cubes is optimal in worst case:

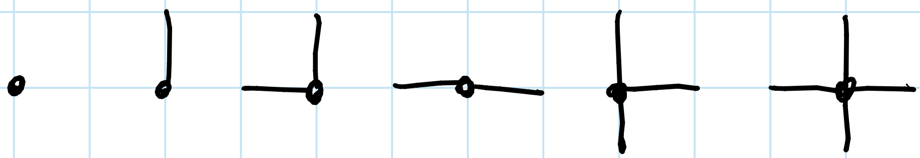
 $1 \times 1 \times n$ needs diameter $\Omega(n)$

- but sometimes can do better:

Maze folding: [Demaine, Demaine, Ku 2010]

any $n \times n$ orthogonal maze extruded from square can be folded from $\Theta(n) \times \Theta(n)$ square

- constant scale factor! (3 for unit extrusion)
- gadget for each possible vertex:



degree 0 1 2 2 3 4

- designed to have compatible interfaces:
  - ridge for maze edges
  - flat "double pleat" for nonedges
- cut & paste

try it out: http://erikdemaine.org/maze/

Origami design is hard ~ how to formalize?

<u>NP-hard</u> ≈ "computationally intractable"
- if a problem is NP-hard, then there's no efficient algorithm to solve it <u>unless P=NP</u> (famous unsolved problem, worth \$1M+)
- P≠NP ≈ "computers can't simulate lucky guessing, say heads vs. tails, without trying both options"
  ↳ almost everyone beleives it

<u>Examples</u> of NP-hard problems:
- <u>Partition</u>: given n integers, can you split them into two halves of equal sum? (e.g. equalizing teams for a game)
  - actually only hard for exp. large integers: "<u>weakly NP-hard</u>"
- <u>SAT</u>: given Boolean formula (x AND NOT y) OR z can you set the variables x, y, z true/false so that formula is true?

<u>Approach</u>: show e.g. Partition is easier / a special case of your problem: any Partition problem can be converted into a problem of your type
⇒ your problem is NP-hard too

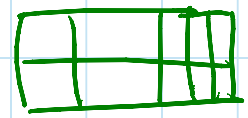
<u>Simple example</u>: (from Problem Session 1)

given single-vertex hinge pattern, is some subset of ($>\emptyset$) creases flat foldable? (posed by student after class)

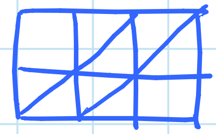
is NP-hard:

- given Partition problem, scale integers uniformly so that their sum = 360°
  → angles of single-vertex crease pattern
- looking at angular travel (Kawasaki-Justin), at each hinge can crease → change direction or not → same direction
  ⇒ can choose $+\theta_i$ or $-\theta_i$ for each i
- must have $\sum_i \pm\theta_i = \emptyset$
  i.e. $\sum_i +\theta_i$'s $= \sum_i -\theta_i$'s
- YES to Partition ⇔ YES to flat-foldable CP

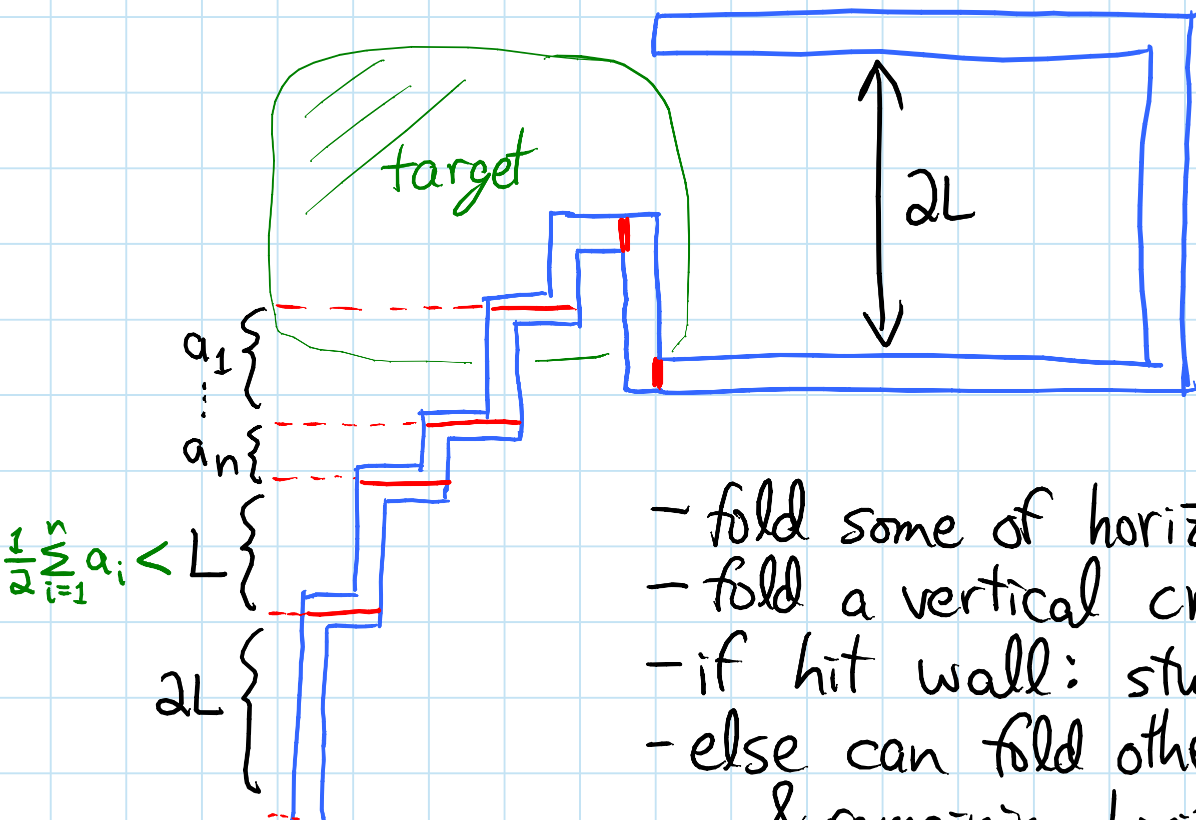
Simple folds: can given crease pattern be folded flat by sequence of simple folds?
- saw how to solve for 1D patterns & 2D orthogonal maps:



NP-hard if we add 45° diagonal creases or allow orthogonal paper



[Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena 2000]
- reduction from Partition (weakly NP-complete)



- fold some of horiz. creases
- fold a vertical crease
- if hit wall: stuck
- else can fold other vertical & remaining horiz. creases

$\square$

Global flat foldability: [Bern & Hayes 1996]

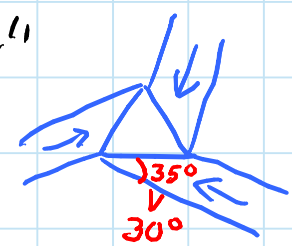
① deciding flat foldability of given crease pattern is strongly NP-hard

② constructing valid layer ordering for given flat-foldable mountain-valley pattern is strongly NP-hard

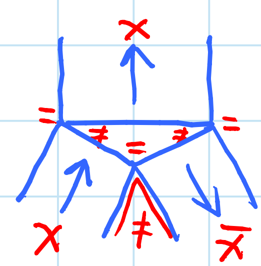
Proof: (①) reduce from all-positive not-all-equal 3-satisfiability: given triples (x_i, x_j, x_k), is there a Boolean assignment to $x_1, x_2, \dots, x_n$ such that no triple is all-true or all-false? (strongly NP-hard, like SAT)

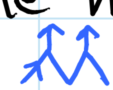
Wire = "pleat" = two close parallel creases
false $\Leftrightarrow$ left mountain

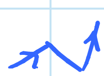
NAE clause = triangular "overtwist"
– can't all fold same way
(twist is borderline)



Reflector splits wire x into two copies, one negated



⇒ split gadget 

& turn gadget  (with noise)

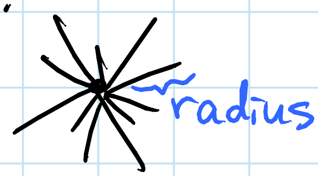
⇒ can connect variable wires to desired clauses

Also need crossover gadgets. □

Disk packing: [Demaine, Fekete, Lang 2010]

can you place n given disks nonoverlapping with centers in given square?

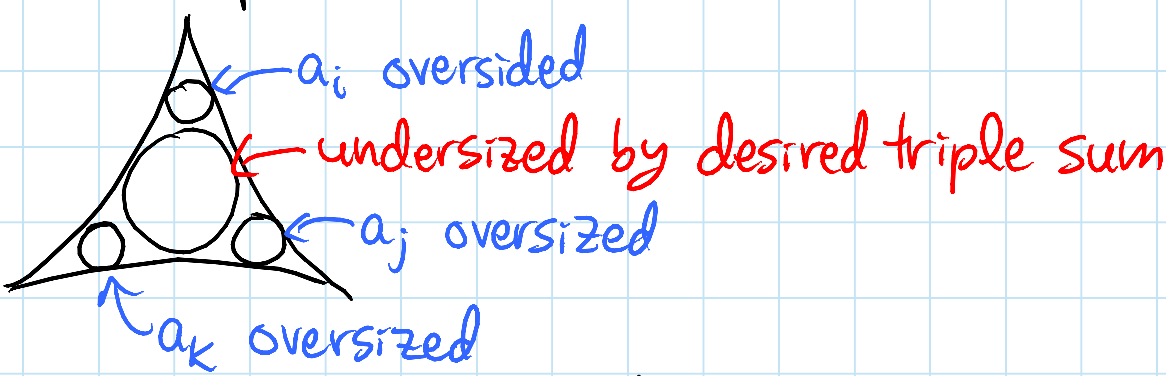
= can you make uniaxial base from given square?



is NP-hard

- reduction from 3-Partition:
  given n integers, can you split them into $n/3$ triples of equal sum?
  - strongly NP-hard ~ integers $= O(n)$, not exp.

- lots of disks to force identical pockets & make all other pockets too small
- within one pocket:



$\Rightarrow$ just fits if 3-partition $\square$