6.849	Class 2	Feb. 15, 2017

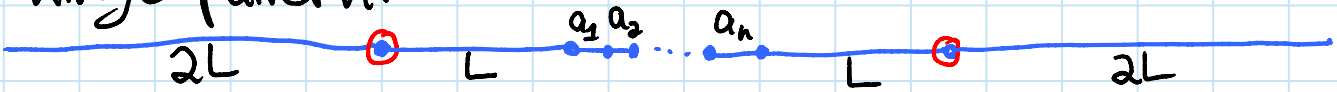
NP-hard ≈ "computationally intractable"
- if a problem is NP-hard, then there's no efficient algorithm to solve it unless P=NP
(famous unsolved problem, worth $1M+)
- P≠NP ≈ "computers can't simulate lucky guessing, say heads vs. tails, without trying both options"
↳ almost everyone believes it

Examples of NP-hard problems:
- SAT: given Boolean formula (x OR NOT y) AND z
can you set the variables x, y, z true/false so that formula is true?
- Partition: given n integers, can you split them into two halves of equal sum?
(e.g. equalizing teams for a game)
  - weakly NP-hard: hard only when integers are exponential in n
  (still polynomial number of bits)
- 3-Partition: given n integers, can you split them into $n/3$ triples of equal sum?
  - strongly NP-hard: hard even when integers are polynomial in n

Approach: show e.g. Partition is easier / a special case of your problem: any Partition problem can be converted into a problem of your type
⇒ your problem is NP-hard too

Simple example: Ruler Folding
- given 1D hinge pattern
  crease pattern where all creases are optional
- given length bound M
- decide: is there a flat folding of length $\leq M$

is weakly NP-hard:
- given Partition problem $a_1, a_2, \ldots, a_n$
- choose $L > a_1 + a_2 + \cdots + a_n$ "HUGE"
- hinge pattern:



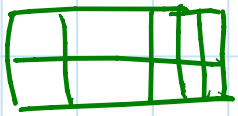
- target length $M = 2L$ ⇒ forced folds

- intended folding:
- looking at travel in 1D:
  at each hinge can crease → change direction
  or not → same direction

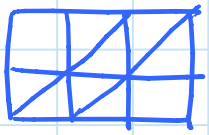
⇒ can choose $+a_i$ or $-a_i$ for each i
- must have $\sum_i \pm a_i = 0$
  i.e. $\sum_i +a_i = -\sum_i -a_i$'s
- YES to Partition ⇔ YES to Ruler Folding □

Simple folds: can given crease pattern be folded flat by sequence of simple folds?

- saw how to solve for 1D patterns & 2D orthogonal maps:

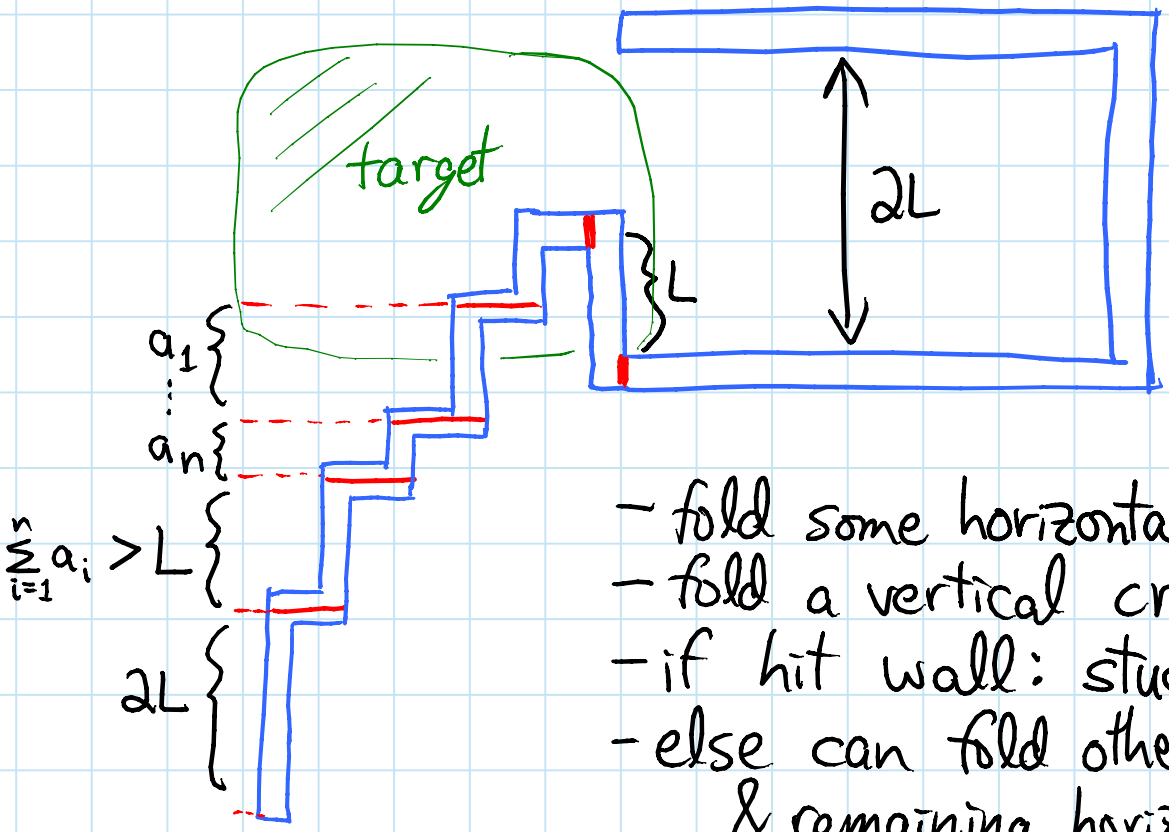


Weakly NP-hard if we add 45° diagonal creases or allow orthogonal paper



[Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena 2000]

- reduction from Partition



- fold some horizontal creases
- fold a vertical crease
- if hit wall: stuck
- else can fold other vertical & remaining horiz. creases

□

Strongly NP-hard by reduction from 3-Partition

[Akitaya, Demaine, Ku 2016]