

6.849

Class 20

Nov. 20, 2012

- Discussion of class as a whole & experimental split into video lectures + live classes

My perspective:

- you asked great questions, which shaped most of classes (lucky?)
- "true" interactivity mostly with few physical exercises (folding, etc.)
- old videos mostly good, >1x great (faster, potentially more focus)
- new videos add lots of complementary stuff
- most students seemed to follow schedule
- I probably needed more lead time

Questions for future:

- 5 hrs./week too much? doesn't scale
- more group activities?
- project-focused class?
- open-problem-focused class?
- in-person lectures better?
- or no fixed-time meetings?

o Equilateral vs. equiangular vs. obtuse
locked 3D chains:

① universal joints:

- OPEN: equilateral $\Rightarrow$ not locked?
- equiangular & obtuse don't seem too relevant here... not necc. preserved

② fixed-angle joints:

- OPEN: equilateral, equiangular, & obtuse $\Rightarrow$ not locked?
- equilateral + equiangular:
conjecture locked "crossed legs"
- equiangular + obtuse:
subdivided knitting needles lock
- equilateral + obtuse:
can simulate long bars $\Rightarrow$ lock

o Why model ribosome as a cone?

- Because it's roughly a halfspace

e.g. [Nissen, Hansen, Ban, Moore, Steitz - *Science* 2000]

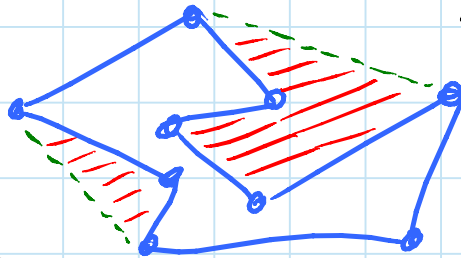
Nobel Prize in Chemistry, 2009 $\rightarrow$

- HP protein folding NP-hardness:
 - 3D cube lattice [Berger & Leighton 1998]
reduces from bin packing
 - 2D square lattice [Crescenzi, Goldman, Papadimitriou, Piccolboni, Yannakakis 1998]
reduces from max.-degree-4 Hamiltonicity

- Flattening is strongly NP-hard
[Demaine & Eisenstat - WADS 2011]

- Flips: (see 2010 L21 for more details)

Pocket of 2D polygon = region outside polygon
& inside convex hull



Pocket lid = convex-hull edge

Flip = reflect pocket through its lid
= rotate 180° through 3D around the lid

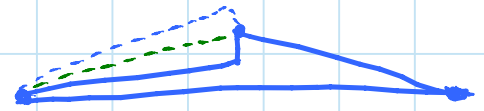
- avoids self-intersection (line of support)
- increases area

"Erdős-Nagy" Theorem: [posed by Erdős 1935]

any polygon always convexifies after finite flips,
no matter how flip sequence is chosen

— but can be arbitrarily many:

[Joss & Shannon 1973]

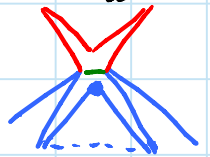


— **OPEN**: bound # flips in n & $r = \text{max. dist.} / \text{min. dist.}$
— pseudopolynomial? [Overmars 1998]

"Proofs" of Erdős-Nagy Theorem:

[Demaine, Gassend, O'Rourke, Toussaint 2007]

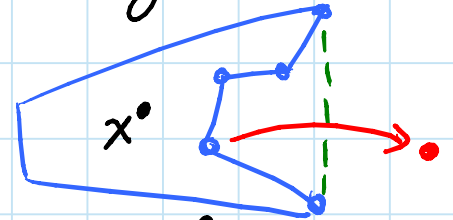
- knowledge
- Nagy 1939 — flawed: " $P^0 \subseteq C^0 \subseteq P^1 \subseteq C^1 \subseteq \dots$ "
(used to "prove" limit polygon convex)
 - Reshetnyak 1957 — correct (though somewhat imprecise)
 - Yusepov 1957 — flawed: "limit convex else flip"
& more subtle error
 - Bing & Kazarinoff 1959 — correct (though somewhat terse)
 - Wegner 1993 — flawed: "move vertex $\Rightarrow$ increase area
by incident Δ "



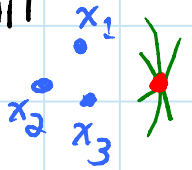
- all
- all
- all
- Grünbaum 1995 — omission: why limit polygon is convex
 - Toussaint 1999/2005 — flawed: "limit convex else flip"
 - Demaine, Gassend, O'Rourke, Toussaint 2008 — generalization to self-crossing
assuming no "hairpins":

Proof of "Erdős-Nagy" Theorem: [Bing & Kazarinoff 1959]
consider an infinite flip sequence & [DGT 2006]

- ① distance from a vertex to fixed point x inside the polygon (remains so) only increases
– pocket lid is Voronoi diagram of old & new



- ② each vertex approaches a unique limit
– apply ① to three noncollinear points x_1, x_2, x_3 inside the polygon
– distances from vertex $\leq$ perimeter of polygon/2
 $\Rightarrow$ distances converge
 $\Rightarrow$ vertex approaches intersection of 3 circles



- ③ turn angle at each vertex converges
– by ②, 3 vertices defining the angle converge
– by ①, vertices do not get closer to each other
– rest by continuity

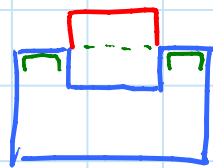
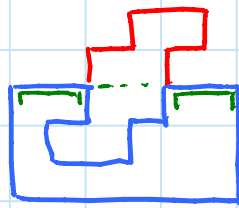
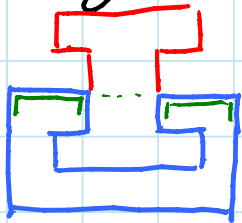
- ④ vertex moves infinitely $\Rightarrow$ asymptotically flat
– each move negates sign of turn angle $\Rightarrow \rightarrow \emptyset$

- ⑤ contradiction
– eventually asymptotically pointed vxs. stop moving
 $\Rightarrow$ attain limit convex hull, but about to flip! $\square$

- Flipturn: rotate pocket 180° in 2D around lid midpoint
- at most $n!$ configurations [Joss & Shannon 1973]
 - always $O(n^2)$ flipturns [Aichholzer et al. 2002; Ahn et al. 2000 (diff. model)]
 - sometimes $\Omega(n^2)$ flipturns [Biedl 2004]
 - final polygon & location determined
 - NP-hard to find longest flipturn sequence
 - OPEN: finding shortest flipturn sequence? [Aichholzer et al. 2002]

Orthogonal polygons: $< n$ flipturns

- count brackets:  &  (polygon interior on either side)
- allow overlap  $\Rightarrow \leq n$ brackets
- claim # brackets never decreases
- (13-case analysis)
- orthogonal flipturn kills two brackets:



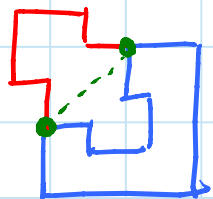
$\Rightarrow \leq n/2$ orthogonal flipturns

- diagonal flipturn kills two vertices:

$\Rightarrow < n/2$ diagonal flipturns

$\Rightarrow < n$ total

□

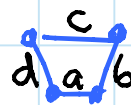


OPEN: $n - O(1)$ flipturns ever possible?

- best example requires $\frac{5}{6}n - O(1)$

Deflation: inverse of flip (avoiding crossings)

- conjectured finite [Wegner 1993]
- quadrilaterals with $a+c=b+d$ & $a \neq b \neq c \neq d \neq a$ always deflate infinitely



[Fevens, Hernandez, Mesa, Morin, Soss, Toussaint 2001]

- that's all such quads [Ballinger, PhD 2003]
- no pentagon always deflates infinitely

[Demaine, Demaine, Fevens, Mesa, Soss, Souvaine, Taslakian, Toussaint 2007]

- OPEN: $n \geq 6$ gon? (no flat vertices)

Pop: flip on 2 incident edges



[Millet 1994]

- can be forced to introduce crossings $\Rightarrow$ allow
- possible to convexify any polygon in finitely many pops? [Ballinger & Thurston 2001]

- NO: "alternating" polygons can't be

[Dumitrescu & Hilscher 2009]

vertices alternate between x & y axes

Popturns: flipturn on 2 incident edges



- equivalent to pops for equilateral polygons

- can convexify any polygon allowing self-intersection

- characterization of when possible without crossing

[Aloupis et al. 2007]