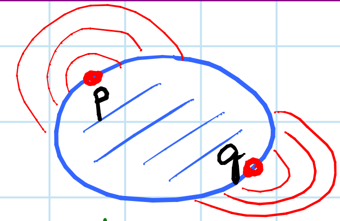
6.849	Class 17	Nov. 8. 2012

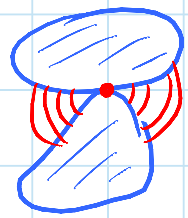
- Pita form: perimeter halving on convex 2D body



  - as performed in L17, but can be smooth

Alexandrov-Pogorelov Theorem: [1950/1973]
every convex metric, topologically a sphere, is realized as the surface of a unique convex 3D body ~> BEYOND POLYHEDRA
- proof by limiting argument + Alexandrov

- D-form: glue together 2 convex 2D bodies of equal perimeter



developable, from a dream... [Tony Wills]

- Seam form: glue together 2D bodies ← flat to satisfy Alexandov-Pogorelov
  - properties: [Demaine & Price - DCG 2009]
    1. = convex hull of seams
    2. creases (other than seams) are line segments with endpoints at (strict) vertices OR tangent to seams

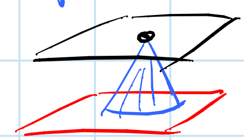
       impossible if 2D bodies convex

  ⇒ D-forms have no seams & pita forms have ≤ 1 crease, pq seam endpoints

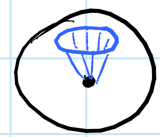
LET'S MAKE SOME D-FORMS!

Proof of 1:

- Minkowski's Theorem: any convex body is the convex hull of its extreme points
  a tangent plane hits just the point
- extreme point can't be locally flat AND convex:



- curvature = area of tangent normals on Gauss sphere

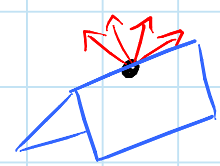


⇒ positive at extreme point of convex body

□

Proof of 2:

- locally flat crease point has range of tangent planes between two extremes



- point can't be extreme by 1

⇒ all tangent planes hit surface

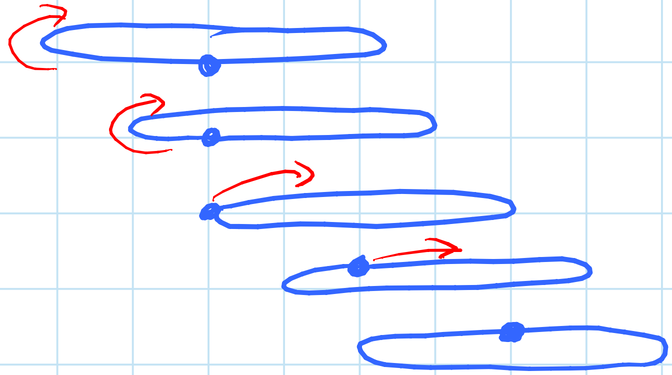
⇒ surface continues along line of intersection, remaining a crease by tangent planes, until not locally flat

- if an endpoint is not a vertex, still zero curvature

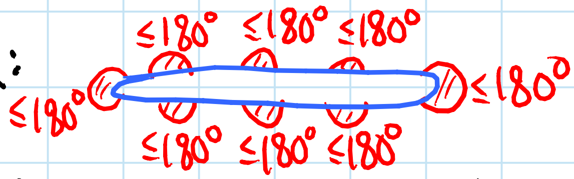
⇒ must be tangent to seam or else get third normal direction ⇒ Gauss area > $\emptyset$

□

- Rolling belt:



  - to work:



(just like a convex body)

- Alexandrov implementation

- Folding nonconvex polyhedra: (see O'Rourke 2010 & Spring 2005)

Burago–Zalgaller Theorem: [1960; 1996] ↗ genus-$\emptyset$ case

every polyhedral metric has an isometric polyhedral realization in 3D.
noncrossing if metric is orientable
or has boundary

- uses Nash's "spiraling perturbations"
- is "strongly corrugated"
- finite # polygons ... but no bound known

$\boxed{\text{OPEN}}$: algorithm to find realization?