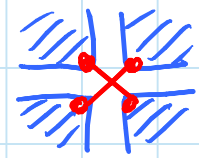
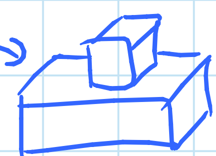
6.849	Class 16	Nov. 6, 2012

- Why are vertex unfoldings (& orthog. unfoldings) "linear"? No love for trees?
  - useful way to avoid overlap
  - trees may help, as in e.g. star/source unf.

- Vertex unfolding (facet path) revisits vertices?
  - YES (but noncrossing: not )
  - no revisit ⇒ e.g. icosahedron impossible
    20 Δs, 12 vertices (if cut all edges)

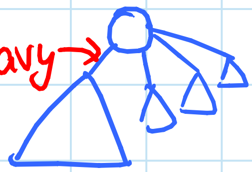
- Topologically convex vertex-ununfoldable:
  [Abel & Demaine — CCCG 2011]
  - only previous vertex ununfoldable: 
  - new example: union of 2 prisms
  - consider light → dark attachment vertex A (all 8 are symmetric)
  - $\alpha > 270°$, $\beta = 90°$, $\alpha + \gamma = 360°$

  ⇒ $\alpha + \beta > 360°$; must join α & γ, uniquely
  - prisms long enough ⇒ overlap
  - bent prisms ⇒ overlap just from $\alpha + \beta'$ & $\alpha + \gamma' > 360°$

$\boxed{\text{OPEN}}$: convex-faced vertex-ununfoldable?

- Unfolding orthogonal polyhedra (genus ∅) with <u>QUADRATIC REFINEMENT</u>: $O(n^2)$
  [Damian, Demaine, Flatland – G&C 2012]
  - <u>review</u>: exponential refinement
    [Damian, Flatland, O'Rourke – G&C 2007]
  - <u>trouble</u>: double worst-case child
    ⇒ exponential for deep tree

  - <u>idea 1</u>: <u>heavy-light decomp.</u>:
    [Sleator & Tarjan – JCSS 1983]
    - <u>heavy child</u> = child with $> 1/2$ the "<u>weight</u>"
      ↳ # descendants



    ⇒ ≤ 1 heavy child of each node
    ⇒ heavy <u>edges</u> form heavy <u>paths</u>
    - <u>light child</u> has $\leq 1/2$ the weight
    ⇒ light <u>depth</u> of any node $\leq \lg n$
    ⇒ can afford to recursively double light children; must be efficient on heavy paths
  - <u>idea 2</u>: last child visited only once!
    ⇒ arrange for it to be heavy child
    - cases: heavy child on back / front
    - light children visited up to 4 times
    ⇒ <u>recurrence</u>: $R(n) = \max_i \{ \underbrace{R(n-1)}_{\text{heavy}}, \underbrace{4 \cdot R(n_i)}_{\text{light}} \}$
    $\leq 4 \cdot R(n/2) = O(n^2)$

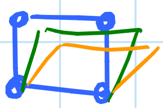
- Practical? probably not

  - $\boxed{\text{OPEN}}$: grid unfolding orthogonal polyh.?
    $\Omega(n)$ refinement necessary?

- $\boxed{\text{OPEN}}$: other lattices?
  - skewed cubes
  - hexagonal → same approach should work?

  c-oriented?

- Isn't Cauchy obvious? no?

  - false in 2D: 

  - flexible nonconvex polyhedra
    [Connelly; Steffen]