6.849	Class 14	Oct. 25, 2012

- **Hinged dissection software:** just specific examples

  $\boxed{\text{PROJECT}}$: hinged dissection animator
  - implement slender adornments (refinement + expansive motion)
  - implement general algorithm?
  - implement polyform algorithm

  $\boxed{\text{PROJECT}}$: design elegant hinged dissections

  Polyform = n copies of one shape glued together along corresp. edges

  Inductive construction: [Demaine, Demaine, Eppstein, Frederickson, Friedman 2005]
  - base case: hinge-dissect 1 copy such that every edge has incident hinge
  - step: take spanning tree of copies
    remove leaf copy
    induct on n−1 remaining copies
    rotate base case to meet them
    reconnect ~ get same hinging

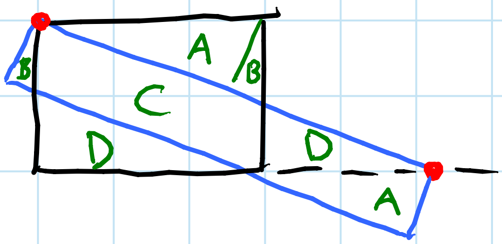
  ⇒ folded states (use slender for motion)

  Also: poly△ → poly□, etc.

3D [Demaine, Demaine, Lindy, Souvaine 2005]

Physical:
- in liquid
- DNA
- Macrobot/Decibot
- related: reconfigurable robots

- Rectangle → rectangle [Montucla 1778]
  - superposing strips method



  - same method for Dudeney's $\Delta \to \square$
  - more stable table [Frederickson 2008]

$\boxed{\text{PROJECT}}$: build reconfigurable furniture

- # pieces doubles? at least, in worst case

○ <u>Pseudopolynomial:</u>

say integer

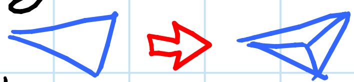
if polygon vertices lie on common grid,

\# pieces = poly(n, r)

↳ # grid positions = $\frac{\text{size}}{\text{cell size}}$

– <u>idea:</u> ensure constant-depth recursion

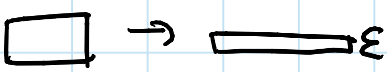
① triangulate polygons <u>with</u> grid vertices
  ⇒ matching Δ areas of ½ [Pick's Theorem]

② chainify 
  ⇒ vertices on $\frac{1}{3}$ grid

③ fix which vertices connect which Δs
  by <u>only</u> modifying parent in subtree move

④ Δ → Δ by overlaying 3 constructions:

Ⓐ Δ →  ← actually done last

Ⓑ 

Ⓒ 

FedEx

... using <u>pseudocuts</u>

↳ simulate cut overlays

- 3D dissection:
  - volumes must match
  - insufficient by Dehn's solution [1901] to Hilbert's third Problem [1900]
  - Dehn invariants must match:

$$\sum_{\text{edge } e} \ell(e) \otimes \left[\theta(e) + \underbrace{\mathbb{Q}\cdot\pi}\right]$$

ignore added rational multiples of π ⇒ "irrational part"

tensor product

  - tensor product space: linear combination of pairs $\ell \otimes \theta$ where

$$\ell_1 \otimes \theta + \ell_2 \otimes \theta = (\ell_1 + \ell_2) \otimes \theta$$
$$\ell \otimes \theta_1 + \ell \otimes \theta_2 = \ell \otimes (\theta_1 + \theta_2)$$
$$c(\ell \otimes \theta) = (c\ell) \otimes \theta = \ell \otimes (c\theta) \quad \forall c \in \mathbb{Q}$$

  - Dehn's Theorem: invariant under dissection
    - e.g.: cut edge $(\ell_1 + \ell_2) \otimes \theta \to \ell_1 \otimes \theta + \ell_2 \otimes \theta$
      slice angle $\ell \otimes (\theta_1 + \theta_2) \to \ell \otimes \theta_1 + \ell \otimes \theta_2$
    - ⇒ no dissection of cube → regular tetrahedron

$$12(1 \otimes \underbrace{90°}_{=0}) \qquad 6(2.04\ldots \otimes \underbrace{70.5288°\ldots}_{\arccos(1/3)})$$

- 3D dissection exists $\Leftrightarrow$ volumes & Dehn Invariants match [Sydler 1965]
- ditto in 4D [Jessen 1968]

$\boxed{\text{OPEN}}$: 5D & higher?

$\boxed{\text{OPEN}}$: efficient algorithm to check Dehn match
- decidable [Kreinovich–Geomb. 2008]

$\boxed{\text{OPEN}}$: algorithm to find dissection

- refinement into hinged dissection still works [Abel et al.]