| 6.849 | Class 2 | Sept. 11, 2012 |
|---|---|---|

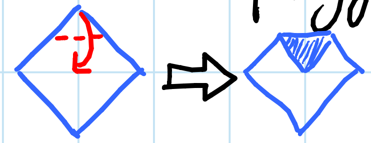
- Funny comments
- Positive comments
  "such a powerful theorem, so quickly"

- Folding practice: numbers 6, 8, 4, 9!
  - cf. Jason Ku's universal alphabet & Jeannine Mosely's 4-fold alphabet
  - PROJECT: design 4-fold digits

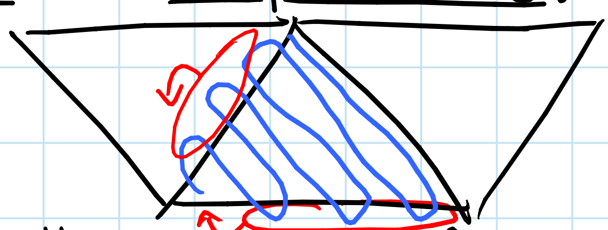
- History: why "silhouette" & "gift wrapping"?

- Practical? strips occasional in origami
  - pretty good $n \times n$ checkerboard design (see L4)

- Pseudopolynomial upper/lower bound? OPEN
  ↳ polynomial in $n$ = # vertices + edges + faces & geometric ratio $r$
  ↳ $(nr)^c$
  - here, $r = \dfrac{\text{max. diameter of face}}{\text{min. altitute of triangle}}$
    & want to bound # folds & aspect ratio
  - upper bound claimed, but not explicit
    - $O(nr)$? $O(n+r)$?
  - presumably a lower bound e.g. $\Omega(n+r)$

- Seam placement
  - convex seam patterns all possible:
    - visit seam polygons in a tour
    - transition increases/decreases width of strip via lg ratio width gadgets & offsets strip to "cover" next polygon
  - some nonconvex possible: 
  - **OPEN**: which?

- Hide gadget via simple folds? (same-layers)



mountains might collide

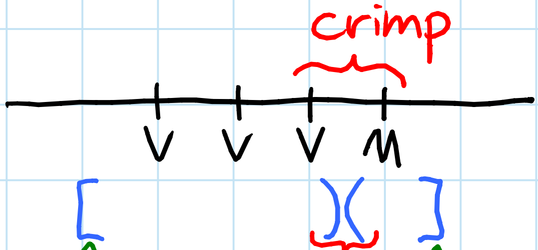
  - silhouette easy: valley fold, not mountain
  - **OPEN**: 2-color pattern?
    - idea: bicolor turn gadget/excess
    - TRY TO SOLVE
  - **OPEN**: convex seam placement

# Simple Folds: [Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena 2000/2004]

- Motivation: bending rigid material

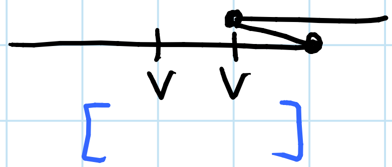
- Definition: single line segment
  $\pm 180°$ rotation
  no collision during motion

- **Example:**



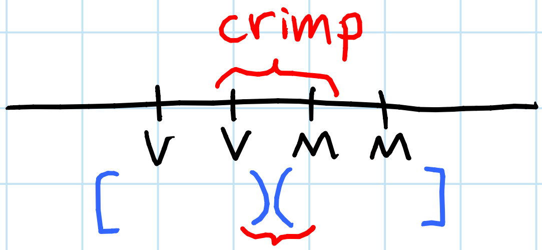

is mingling
(but not "forever")

not left mingling  mingling  not right mingling

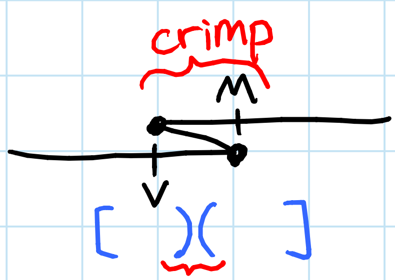



is not mingling
& not flat foldable

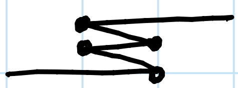
**Another:**



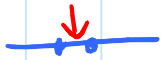

is mingling
& flat foldable




is mingling
& flat foldable



done

- **Algorithm:** (NEW) (covered in C3)
  - search (left to right) for segment that's crimpable  OR end foldable 
  - if none found: STOP ~ not flat foldable
  - else: do fold 
    merge segments $x, y, z \to x-y+z$
    go back one segment (left of $x$)
    continue search

**Correctness:**

- doing fold changes foldability only of adjacent segments

$\Rightarrow$ enough to back up 1 step

**Running time:** $O(n)$ ← # creases

- $\# \text{ right steps} = n + \underbrace{\# \text{ left steps}}_{= \# \text{ folds done}}$

$\leq 2n$

(amortization: charge left steps to fold just done)

- Every mountain-valley pattern can be made flat foldable by adding creases:
  - between consecutive MM's add V & between consecutive VV's add M
  
  $\Rightarrow$ alternating M/V
  
  $\Rightarrow$ flat foldable
  
  (globally smallest segment is crimpable)

- d-dimensional paper

  $\Rightarrow$ (d-1)-dimensional creases
  
  & (d+1)-dimensional ambient space
  - d-dimensional = flat folding