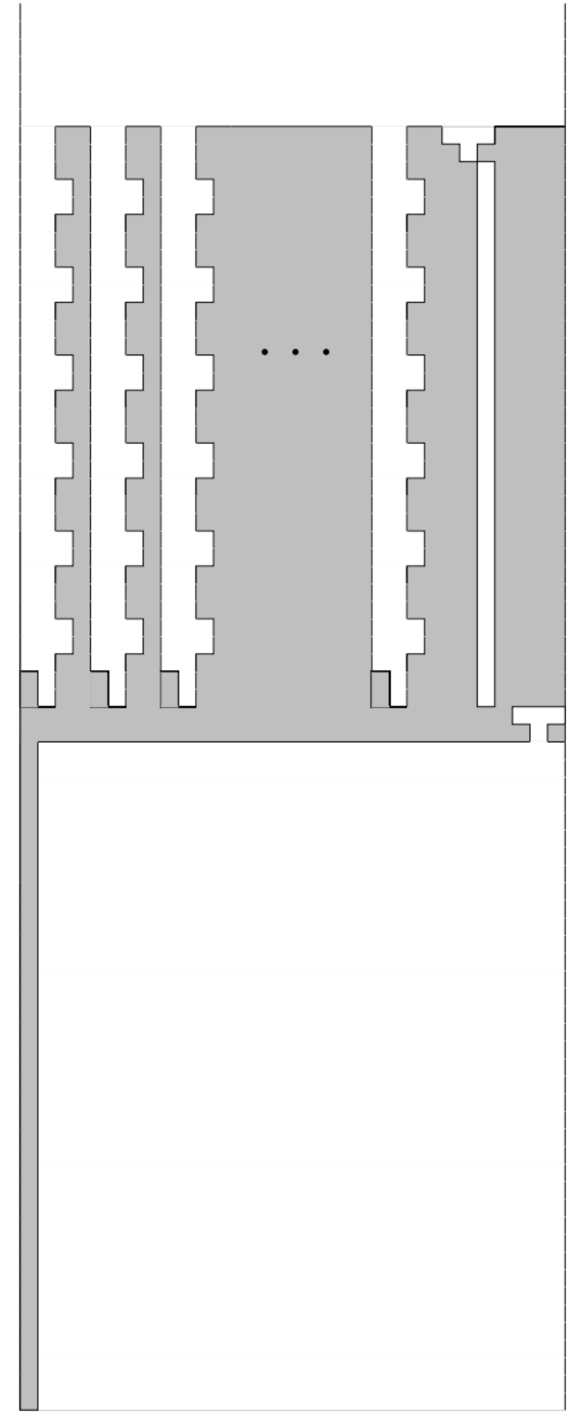
Tetris Hardness of Approximation

[Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell 2003]



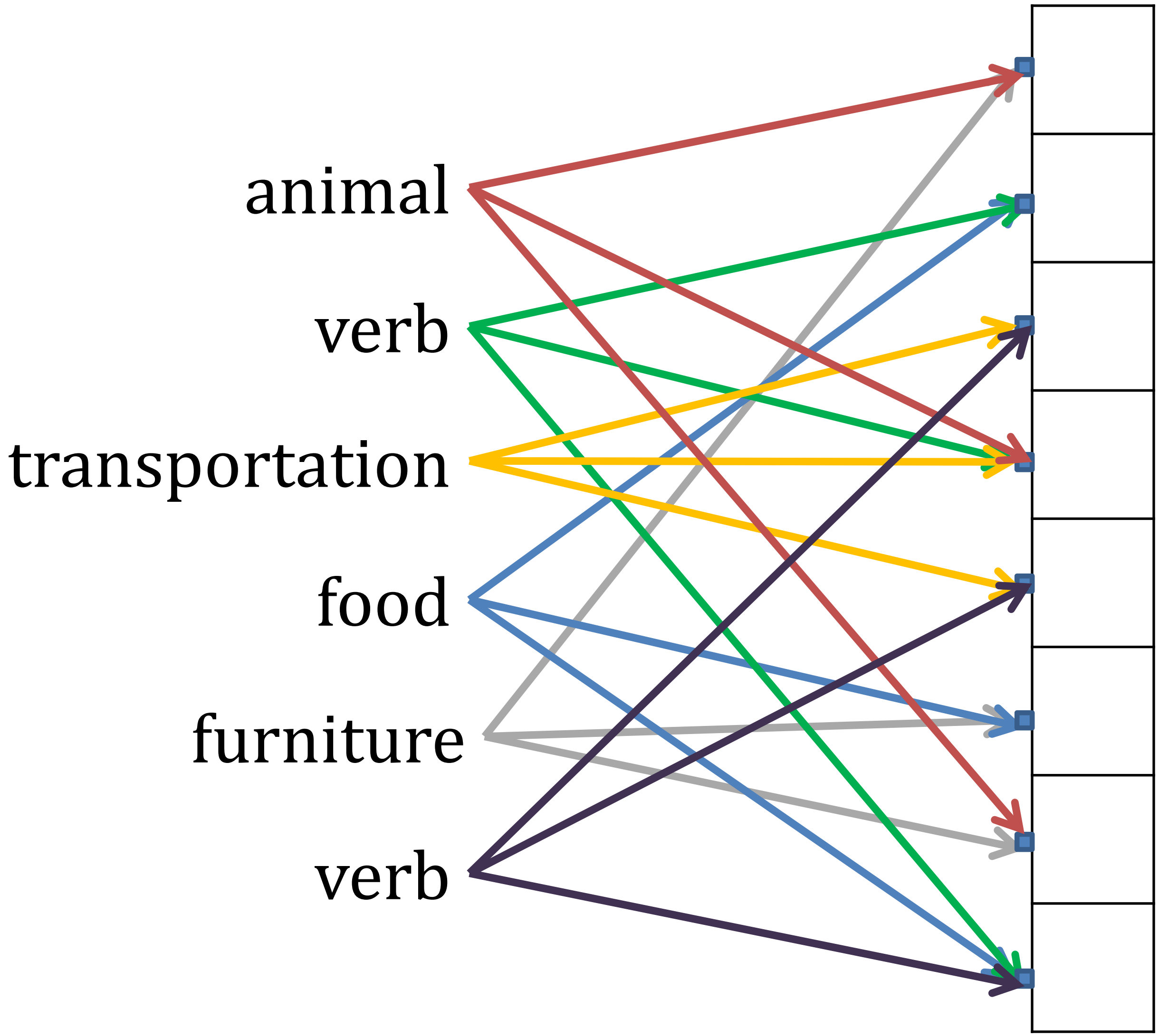
Max E3-X(N)OR-SAT → Max E3SAT

[Håstad 2001]

Max E3SAT:

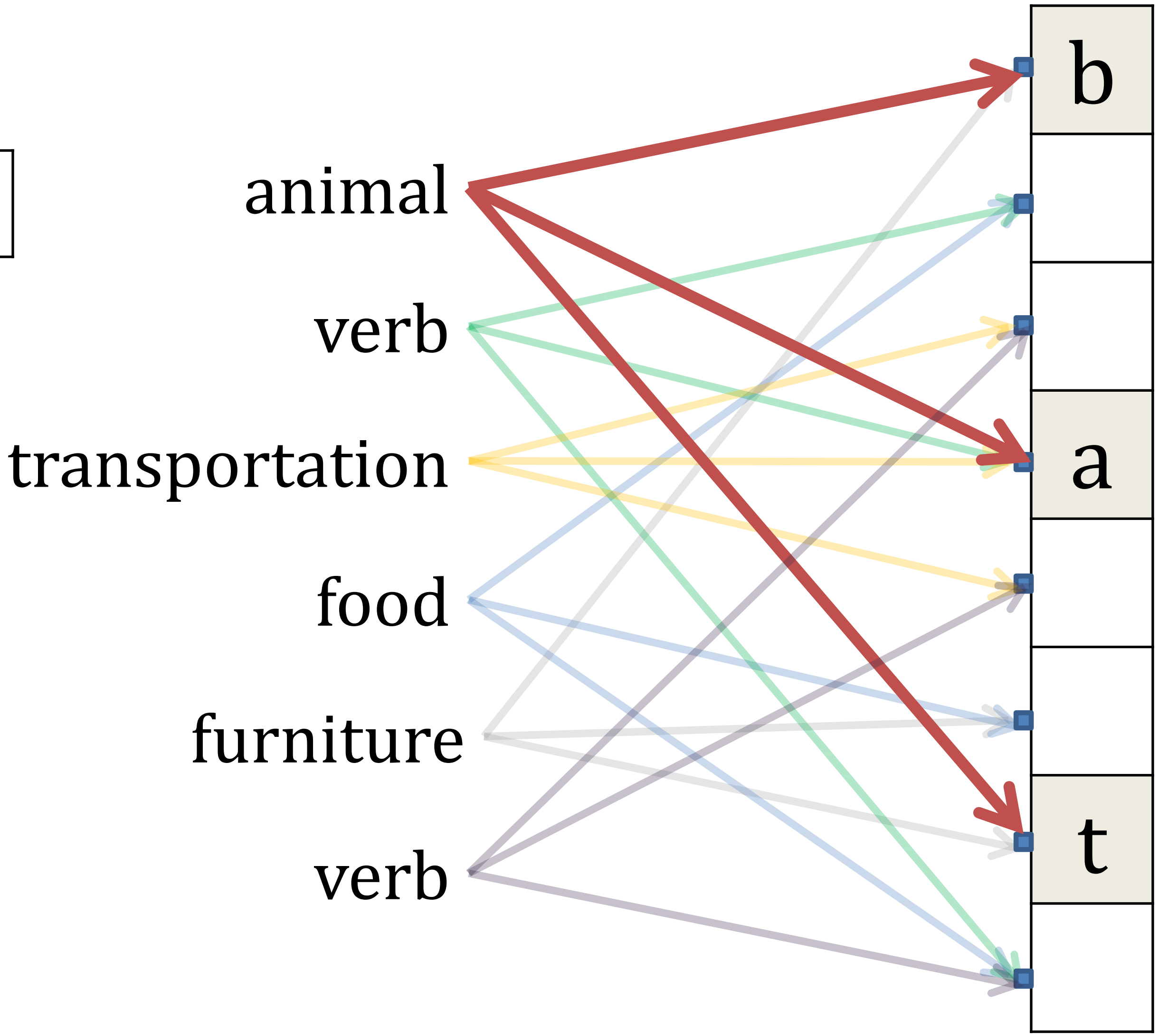
- L-reduction from Max E3-X(N)OR-SAT:
  - $x_i \oplus x_j \oplus x_k = 1 \rightarrow (x_i \vee x_j \vee x_k) \wedge (\bar{x}_i \vee \bar{x}_j \vee x_k) \wedge (x_i \vee \bar{x}_j \vee \bar{x}_k) \wedge (\bar{x}_i \vee x_j \vee \bar{x}_k)$
  - $x_i \oplus x_j \oplus x_k = 0 \rightarrow (\bar{x}_i \vee \bar{x}_j \vee \bar{x}_k) \wedge (x_i \vee x_j \vee \bar{x}_k) \wedge (\bar{x}_i \vee x_j \vee x_k) \wedge (x_i \vee \bar{x}_j \vee x_k)$

Projection Puzzles [Moshkovitz]



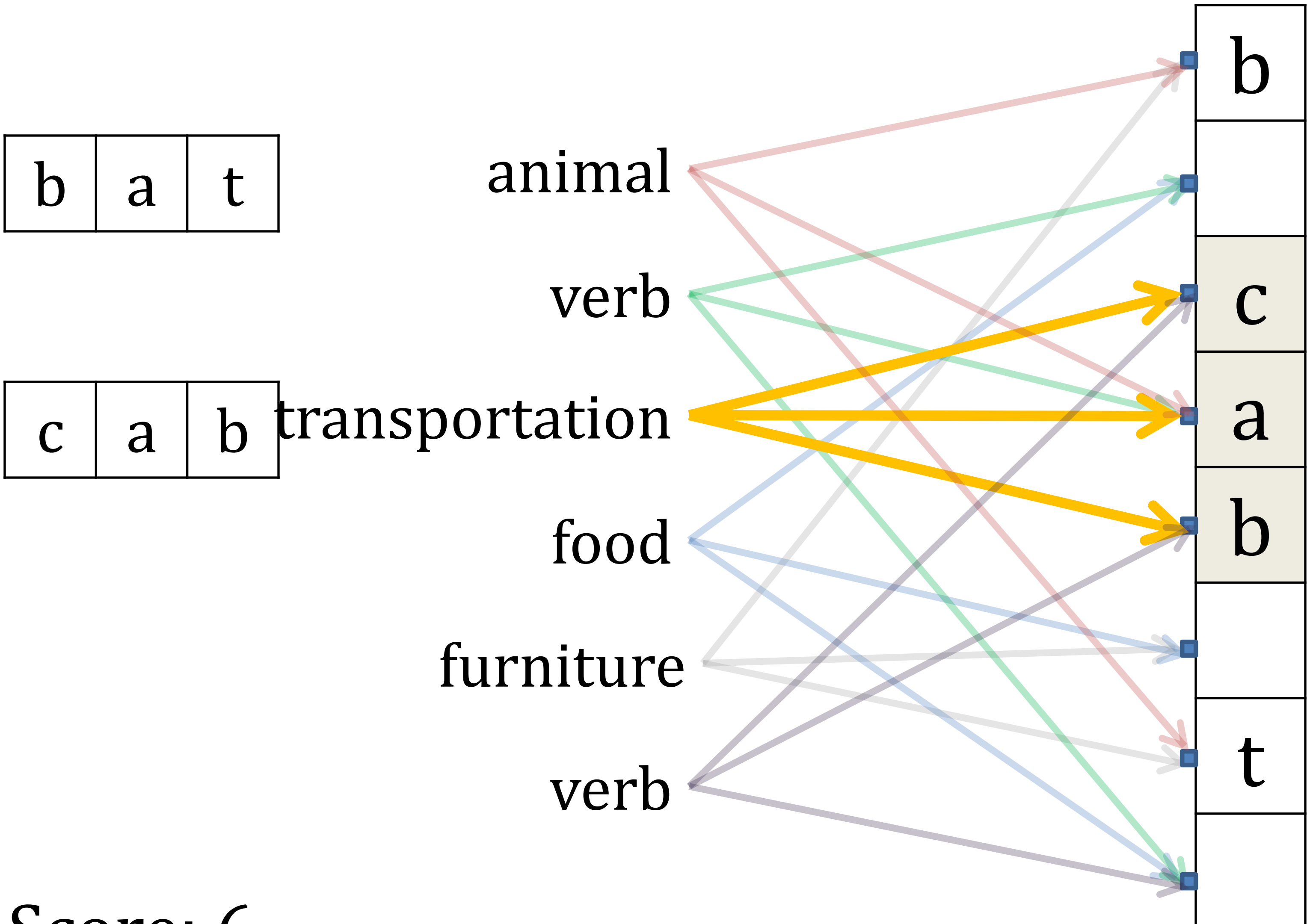
Projection Puzzles [Moshkovitz]

b	a	t



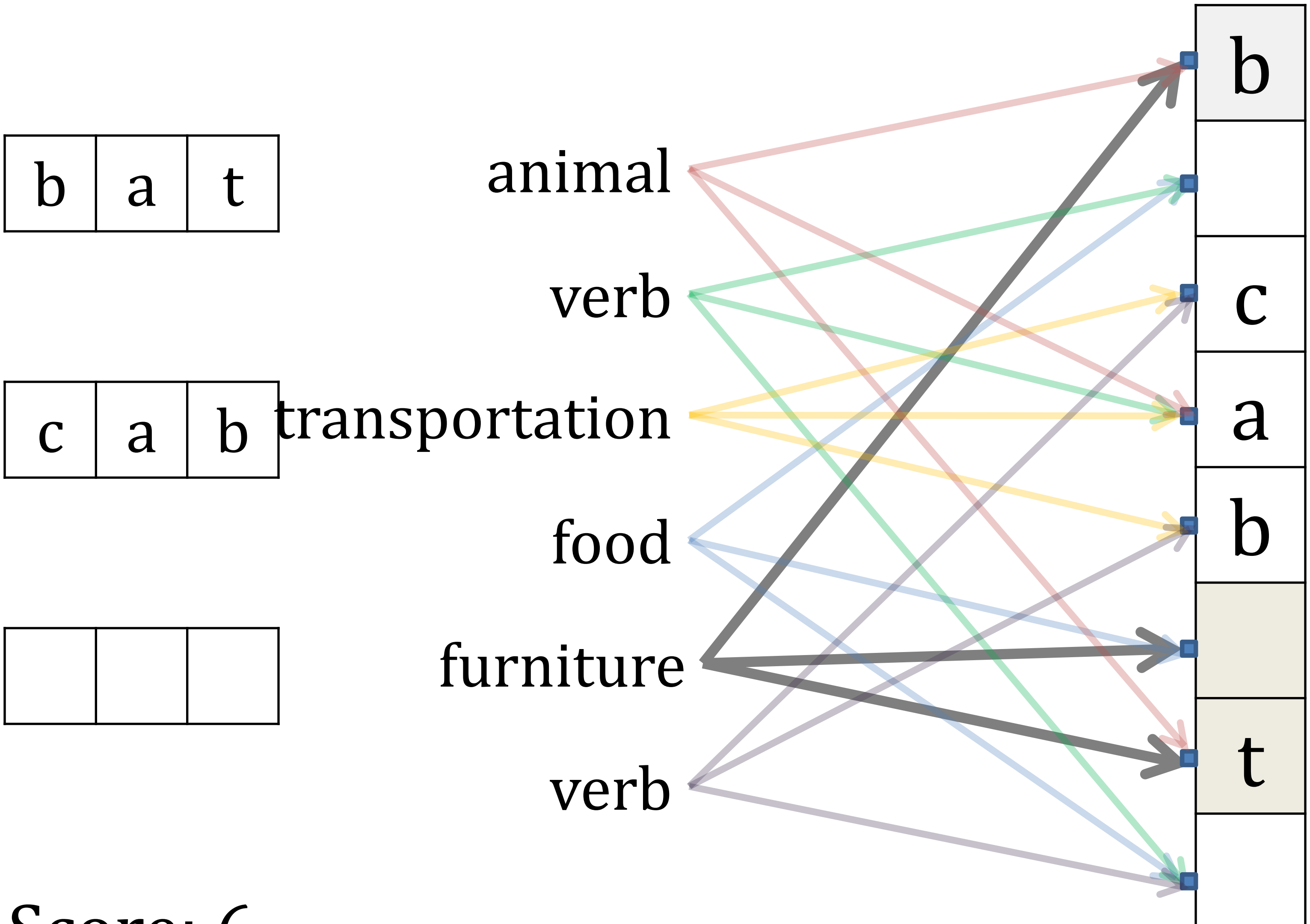
Score: 3

Projection Puzzles [Moshkovitz]



Score: 6

Projection Puzzles [Moshkovitz]



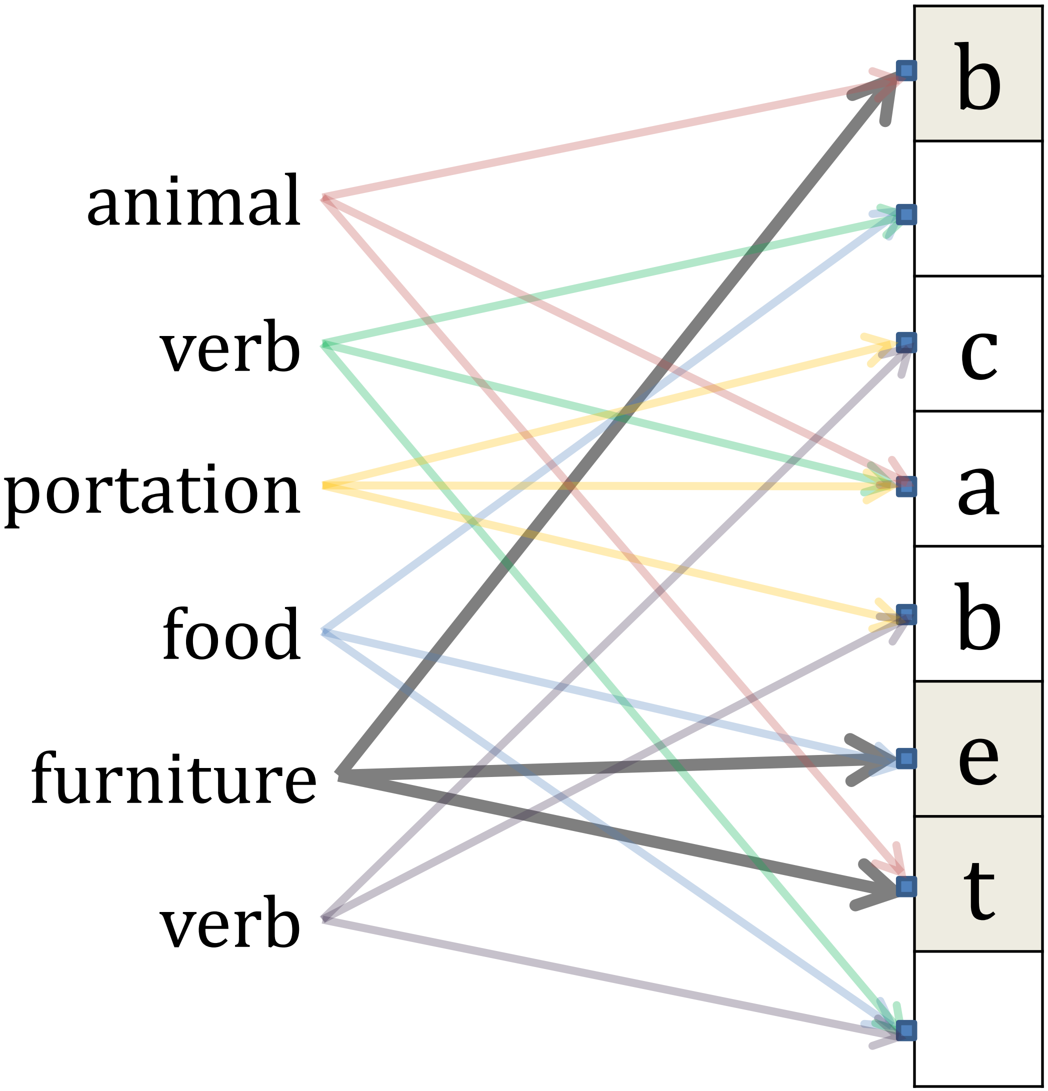
Score: 6

Projection Puzzles [Moshkovitz]

b	a	t

c	a	b

b	e	d



Score: 8

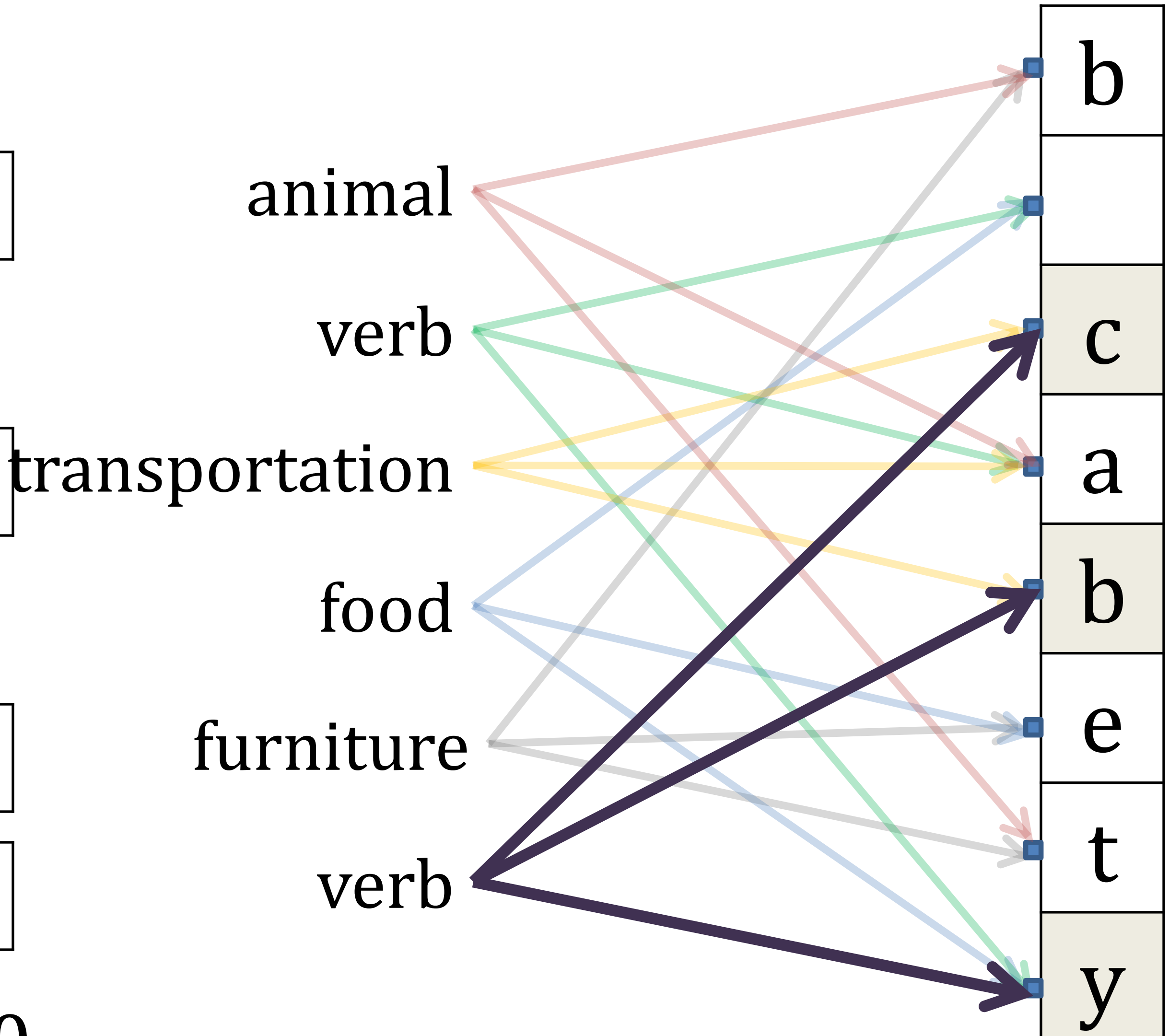
Projection Puzzles [Moshkovitz]

b	a	t

c	a	b

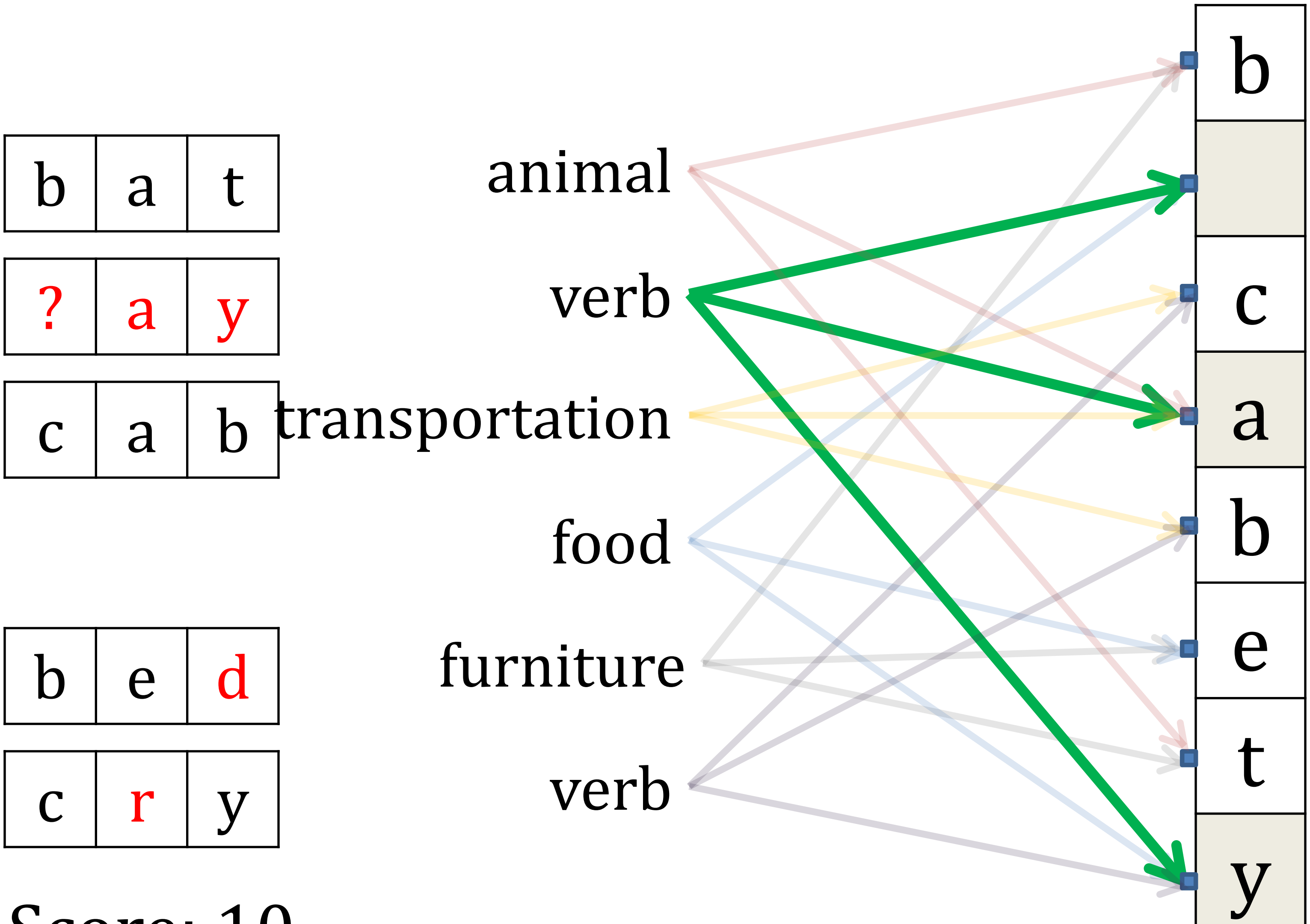
b	e	d

c	r	y



Score: 10

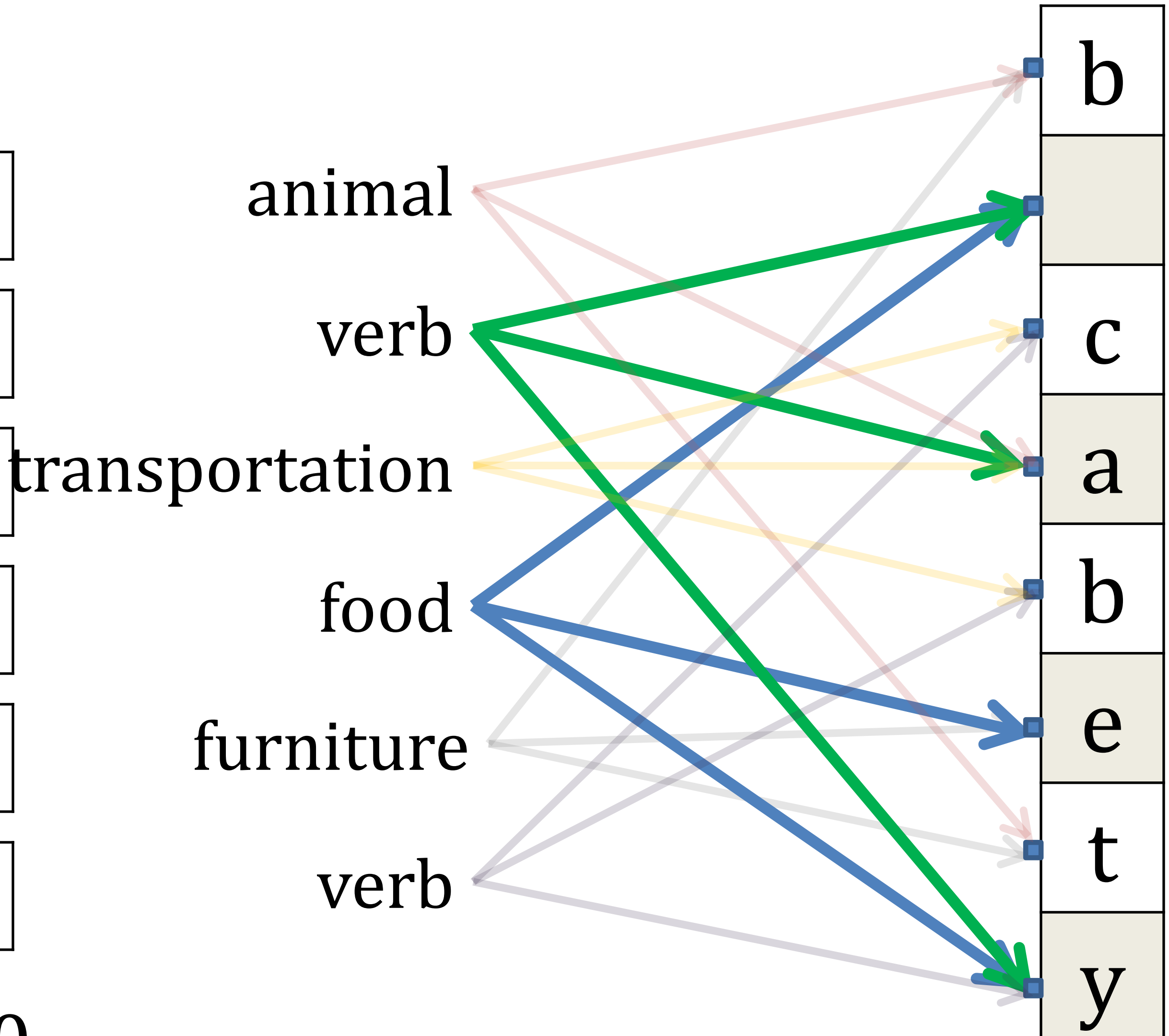
Projection Puzzles [Moshkovitz]



Score: 10

Projection Puzzles [Moshkovitz]

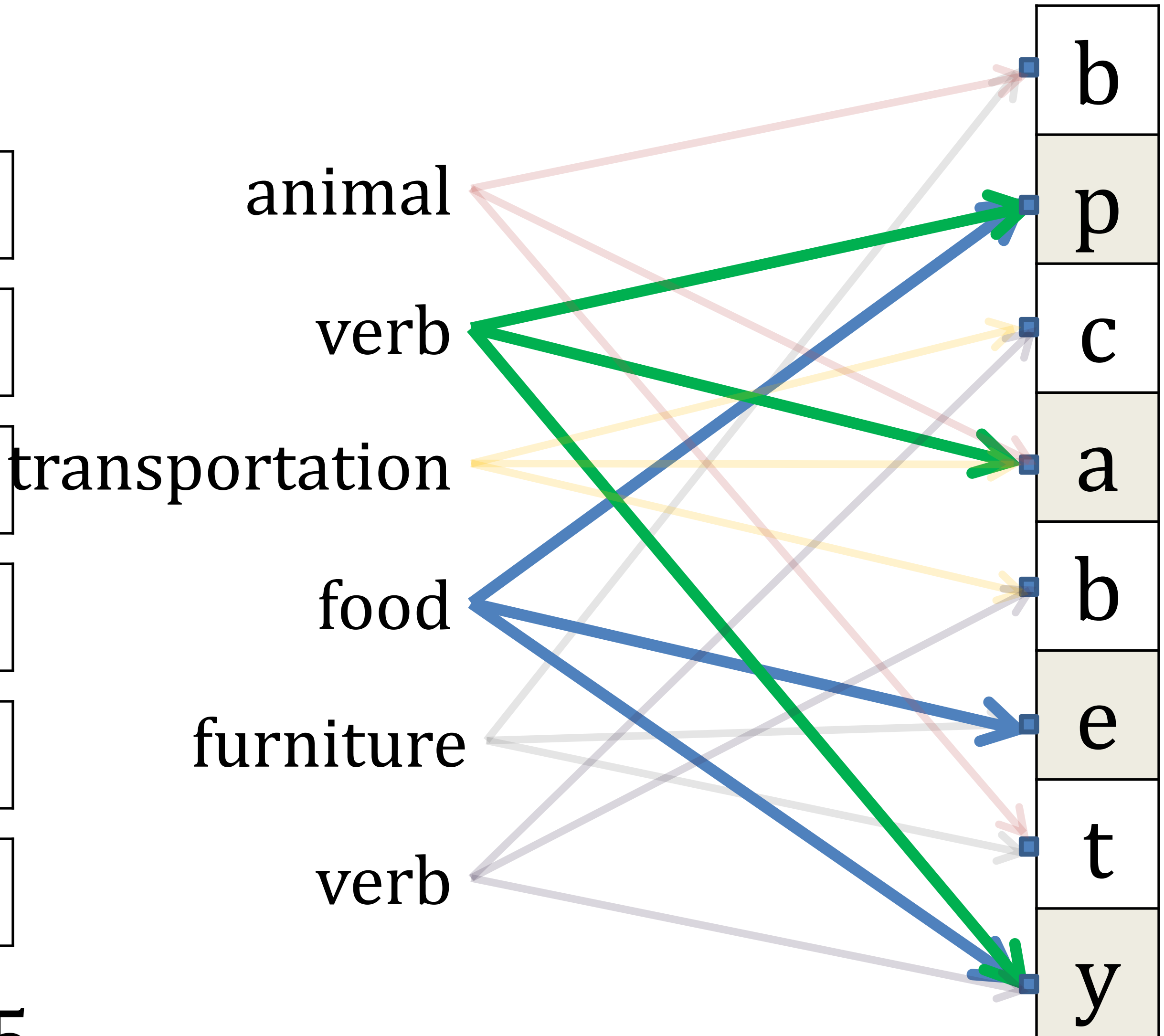
b	a	t
?	a	y
c	a	b
?	e	y
b	e	d
c	r	y



Score: 10

Projection Puzzles [Moshkovitz]

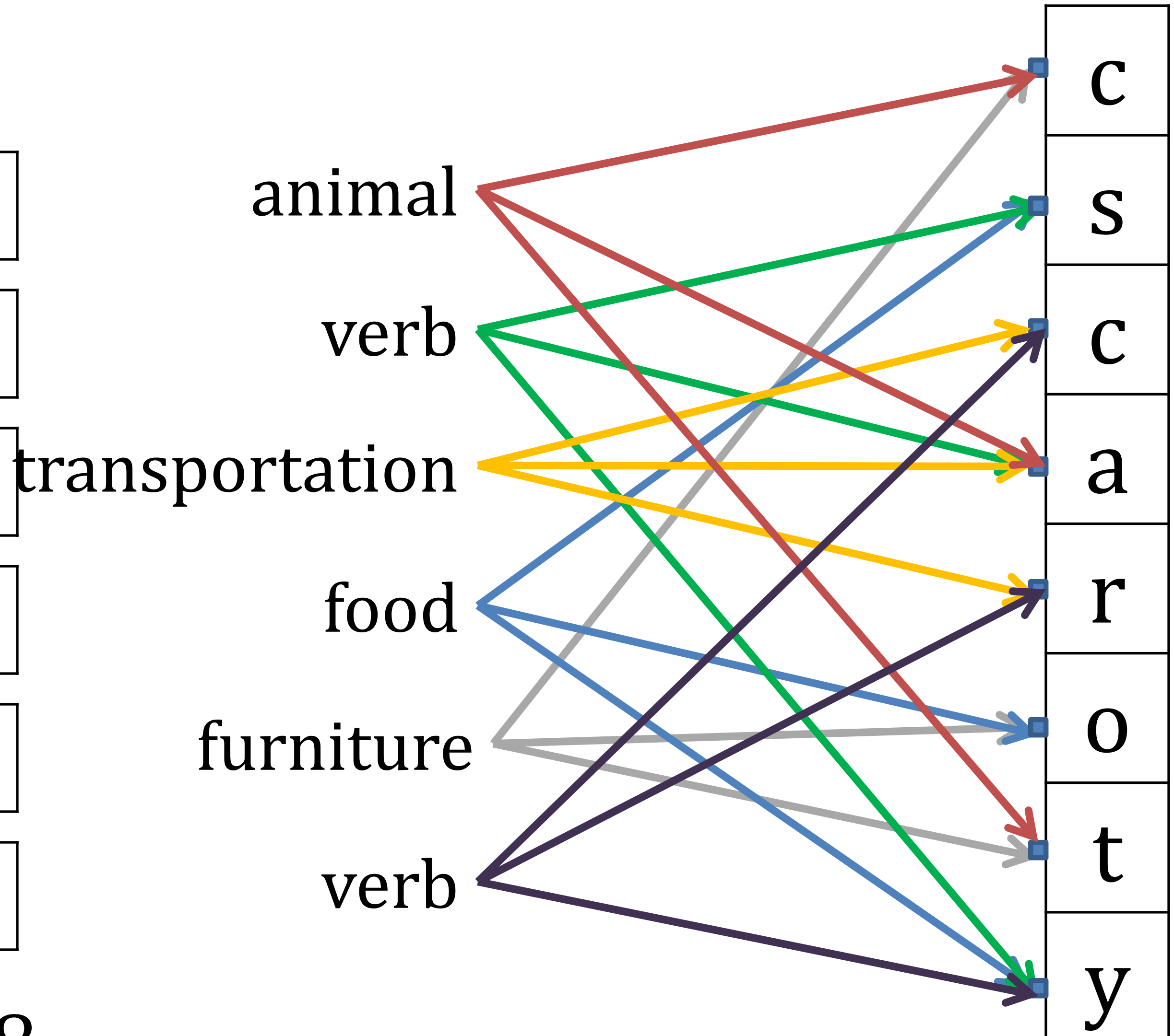
b	a	t
p	a	y
c	a	b
p	e	a
b	e	d
c	r	y



Score: 15

Projection Puzzles [Moshkovitz]

c	a	t
s	a	y
c	a	r
s	o	y
c	o	t
c	r	y



Score: 18

Typical Approximation Factors

Approximation Factor	Minimization Problems	Maximization Problems
$1+\varepsilon$	Planar/H-minor-free/2D … e.g. dominating set	Planar/H-minor-free/2D … e.g. independent set
$\Theta(1)$	Steiner tree, Steiner forest, Traveling Salesman, …	Maximum coverage, Max cut
$\Theta(\log^* n)$	Asymmetric k-center	
$\Theta(\log n)$	Set cover, Dominating set, Node-weighted Steiner tree …	Unique coverage, Domatic number
$\Theta(\log^2 n)$	Group Steiner tree	
$\Omega(\log^2 n) \cap O(n^\varepsilon)$	Directed Steiner tree	
$\Omega\left(2^{\log^{1-\varepsilon} n}\right) \cap O(n^c)$	Label cover (MinRep), $c=\frac{1}{3}$ Directed Steiner forest $c=\frac{4}{5}+\varepsilon$	Label cover (MaxRep)
$\Omega(n^{1-\varepsilon}) \cap \tilde{O}(n)$	Chromatic number	Independent set = clique

Reductions to Steiner Problems

Directed Steiner forest: strict reduction from Min-Rep

require $a_i \to b_j$ path for each superedge (A_i, B_j)

Node-weighted Steiner tree: strict reduction from Set Cover