6.890	Lecture 20	Nov. 13, 2014

↳ actually 11

Bounded team private-information games:

NEXPTIME-complete [Peterson, Reif, Azhar – C&M 2001]

– Dependency QBF (DQBF): [Peterson & Reif – FOCS 1979]

$$\underbrace{\forall X_1 : \forall X_2}_{\text{black player}} : \underbrace{\exists Y_1(X_1)}_{\substack{\text{white 1} \\ \text{only sees } X_1}} : \underbrace{\exists Y_2(X_2)}_{\substack{\text{white player 2} \\ \text{only sees } X_2 \text{ variables}}} : \text{CNF formula}$$

- – can white force a win? (satisfied formula)
- – only one round! (multiple rounds don't help)
- – $\in$ NEXPTIME: guess Y_1 $\underbrace{\forall X_1}_{\text{exponential}}$ & Y_2 $\underbrace{\forall X_2}$

– Bounded Team Private Constraint Logic (TPCL) with 3 players & planar graph

- – moves must be known legal with visible information
- – $\in$ NEXPTIME: guess strategy for all possible visible information (exp. # states)
- – reduction from DQBF
- – first black sets all vars. (white twiddles thumbs)
- – chosen activates → long chain (black threat)
- – white players set their vars.
- – chosens → unlock all → formula activation
- – white wins (just in time) if formula satisfied

Unbounded team private-information games: undecidable [Hearn & Demaine]

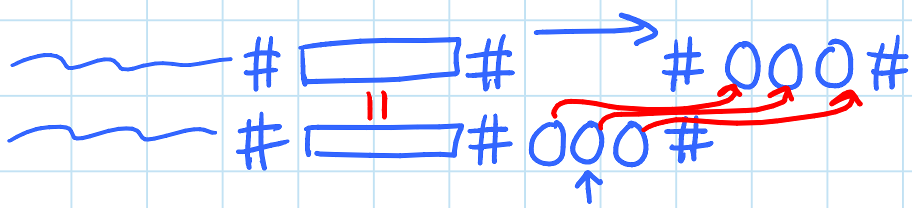
(based on work by Peterson & Reif — FOCS 1979)

Team Computation Game:

- instance = space-k algorithm/Turing machine (memory/tape initially blank)
- black move = run alg./machine for k more steps; output (if any) determines winner; else set $x_1, x_2 \in \{A, B\}$
- white i sees only x_i & can set only m_i
- white i move = set m_i
- does white have a forced win?

- reduction from Halting problem: does this Turing machine ever terminate?
- build O(1)-space algorithm to check white players play valid computation history → halt of the form # state_0 # state_1 # ⋯ # haltstate
- in fact each white player must have in mind 2 pointers A & B into common history
- $x_i = A$ asks for character at A & advance A
- but white players have no idea of other's A/B
- alg. maintains whether W1's x_1 state = W2's x_2 state (identical from # with (x_1, x_2) moves since)

- then if $(x_1, \overline{x_2})$ moves until W1 reports #, → 1 x_1 ahead one
  and if (x_1, x_2) moves then continue,
  then check this W1 state valid transition from W2's
  & vice versa with W1→W2 ↳ O(1) space!
- white strategies must work for all possible
  black moves ⇒ valid computation history



- Team Formula Game:
  - black sets X such that $F(X, X', Y_1, Y_2)$ (else lose)
  - black wins if $G(x)$ ⇕ $F \Rightarrow \neg F'$
  - black sets X' such that $F'(X, X')$ (else lose)
  - white 1 sets Y_1, seeing only Y_1 & $x_1 \in X$
  - white 2 sets Y_2, seeing only Y_2 & $x_2 \in X$
  - standard reduction from Team Computation Game

- (Unbounded) TPCL with 3 players, planar graph

Parallelism & P-completeness:

- book by Greenlaw, Hoover, Ruzzo [Oxford 1995]
  "Limits to Parallel Computation: P-Completeness Theory"

NC (Nick's Class, after Nick Pippinger)
= {problems solvable in $\log^{O(1)} n$ time
using $n^{O(1)}$ processors (PRAM)
i.e. circuit of size $n^{O(1)}$ & depth $\log^{O(1)} n$}

- e.g. sorting: compare all pairs,
  compute rank = sum of <'s
  via binary tree

  } $O(\lg n)$ time on $O(n^2)$ proc.

P-hard = all problems $\in P$ can be reduced via NC algorithm to your problem
Karp-style reduction

$\Rightarrow \notin NC$ if $NC \neq P$

P-complete = $\in P$ + P-hard

Base P-complete problems:

Generic Machine Simulation Problem:
given a sequential algorithm & time bound t written in unary, does it say YES within t?
↳ to make $\in P$ ~ else EXPTIME-complete

Circuit Value Problem (CVP): [Ladner - SIGACT 1975]
given an (acyclic) Boolean circuit & input bits, is the output TRUE?
0 & 1

NAND CVP: just NAND gates
NOR CVP: just NOR gates
Monotone CVP: just AND & OR gates
Alternating monotone CVP: (AMCVP)
input → output path alternates AND/OR, starting & ending with OR
Fanin-2, fanout-2 AMCVP: (AM2CVP)
all gates have in & out degree 2
(allow outputs other than one of interest)
Synchronous AM2CVP: (SAM2CVP)
all inputs to each gate have same depth

Planar CVP: planar circuit [Goldschlager - SIGACT 1977]
- use NAND crossover
- but: planar monotone $\in$ NC [Yang - FOCS 1991]

Reductions: [Greenlaw, Hoover, Ruzzo - book 1995]

- start & end with ORs
- reduce fan out to ≤ 2 (also fanin to ≤ 2)
- make AND & OR alternate
- fanin 1 $\to$ fanin 2
  (preserving alternation & start with OR)
- fanout 1 $\to$ fanout 2
  by duplicating circuit $x \to x$ & x'
  & combining extra outputs
  (preserving alternation & end with OR)
- synchronization: $n = \#$gates
  - $n/2$ copies of circuit
  - ith copy = levels $\underbrace{2i}_{\text{inputs \& ANDs}}$ & $\underbrace{2i+1}_{\text{ORs}}$
    - OR takes inputs from ith copy, sends outputs to $(i+1)$st copy
      (determining ANDs by alternation)
  - AND in 0th copy become 0 input
    $\Rightarrow$ level 0 = inputs
  - inputs fed to ith copy by input gadget
  - output in $n/2$ copy

Bounded DCL:

[Hearn & Demaine]

- edges are active (just flipped) or inactive
- vertex active if its active incoming edges have total weight ≥ 2
- round = reverse unreversed edges pointing to active vertices (& these are the new active edges)

- P-complete for AND, SPLIT, OR graphs (but not necessarily planar)
- reduction from Monotone CVP
- even easier from SAM2CVP

Lexically first maximal independent set:

- as found by greedy algorithm: $\Rightarrow \in P$

  $S = \emptyset$
  for $v = 1, 2, \ldots, |V|$:
  if v not adjacent to S:
  $S = S \cup \{v\}$

- decision question: is $v \in S$?
- P-hard: [Greenlaw, Hoover, Ruzzo - book 1995]
  - reduction from NOR CVP
  - number gates & inputs in topological order
  - drop edge orientations ($\in NC$)
  - add vertex $\emptyset$ connected to all $\emptyset$ inputs

  $\Rightarrow v \in S \Leftrightarrow v = \emptyset$ or gate v outputs true

- computing whether size $\leq k$ also P-complete:
  - reduction from previous problem
  - connect v to $n+1$ new vertices, set $k = n$

  $\Rightarrow$ size $\leq n \Leftrightarrow v \in S$

- gap-producing reduction: $n+1 \rightarrow n^c$

$\Rightarrow n^{1-\varepsilon}$-gap problem is P-complete
$\Rightarrow n^{1-\varepsilon}$-approximation is P-complete

More P-complete problems:

[Greenlaw, Hoover, Ruzzo - book 1995]

- Game of Life: cell (x,y) alive at unary time t?
- 1D cellular automata
- acyclic Generalized Geography
- is point p on kth convex hull of point set?
- multilist ranking: given k lists, is x the kth smallest in the union?
- $a \bmod b_1 \bmod b_2 \cdots \bmod b_n = 0$?
- first fit decreasing bin packing
- LP with coefficients 0 & 1

$\Big\}$ strongly P-complete

- max flow — has fully RNC approx. scheme

$\boxed{\text{OPEN}}$:

- are two numbers relatively prime?
- $a^b \bmod c$
- feasibility of LP with ≤ 2 variables per inequality
- maximum edge-weighted matching
  - pseudo RNC algorithm
- bounded-degree graph isomorphism