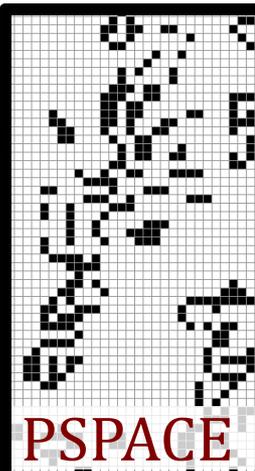


Complexity of Games & Puzzles

unbounded



PSPACE



PSPACE



EXPTIME



Rengo Kriegspiel?

Undecidable

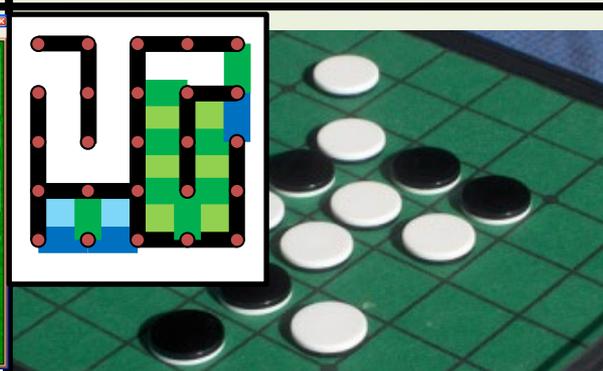
bounded



P



NP



PSPACE



bridge?

NEXPTIME

0 players
(simulation)

1 player
(puzzle)

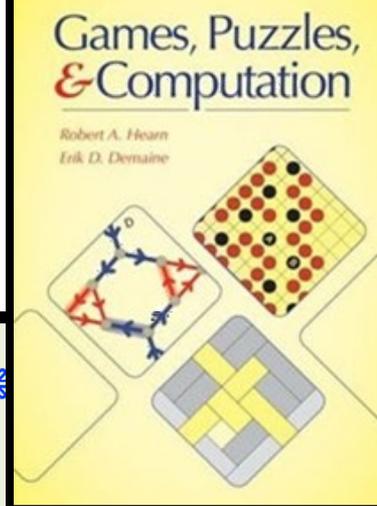
2 players
(game)

team,
imperfect info

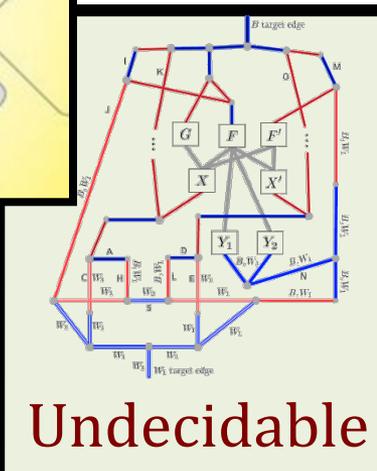
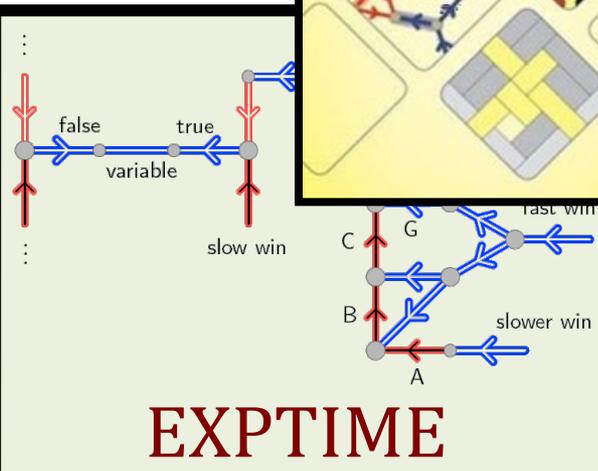
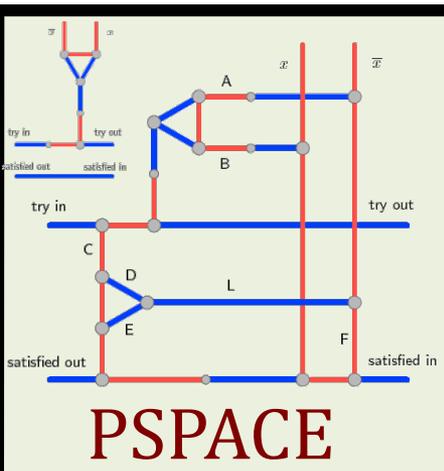
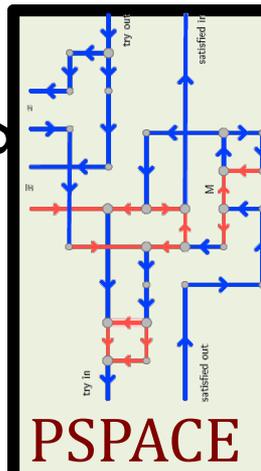


Constraint Logic

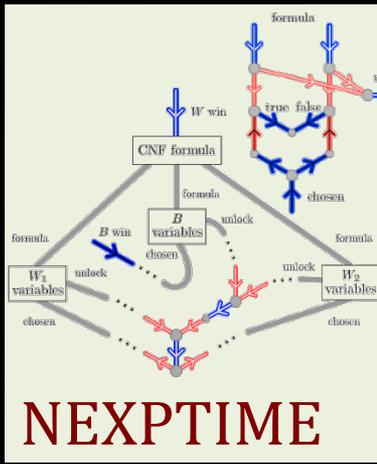
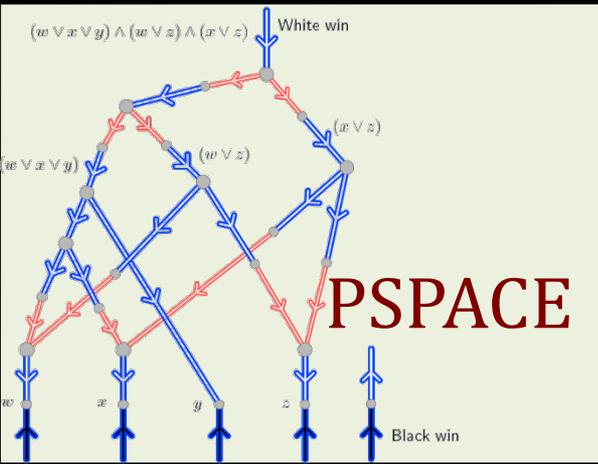
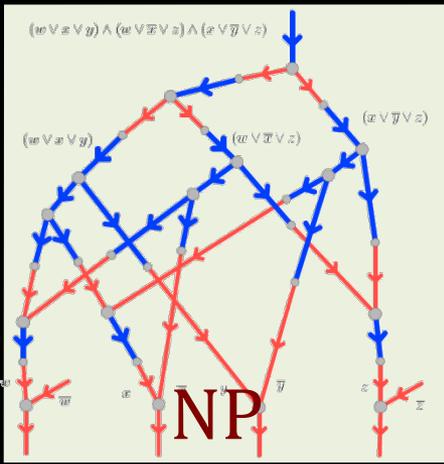
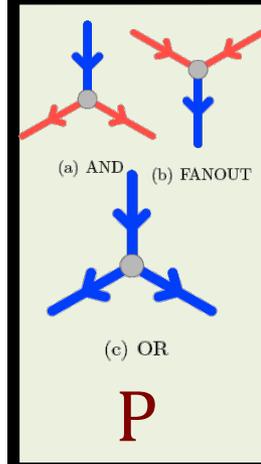
[Hearn & Demaine 2009]



unbounded



bounded



0 players
(simulation)

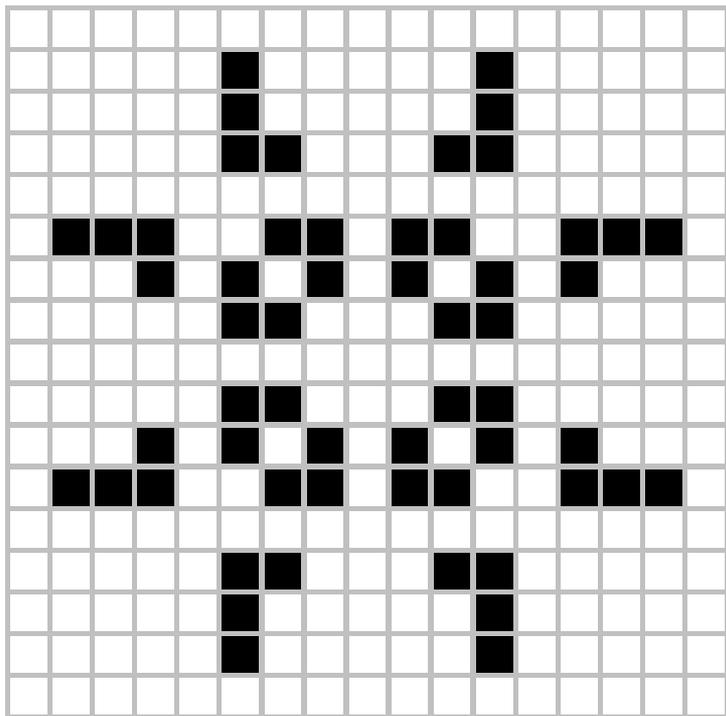
1 player
(puzzle)

2 players
(game)

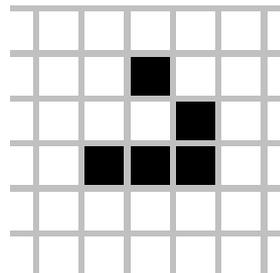
team,
imperfect info



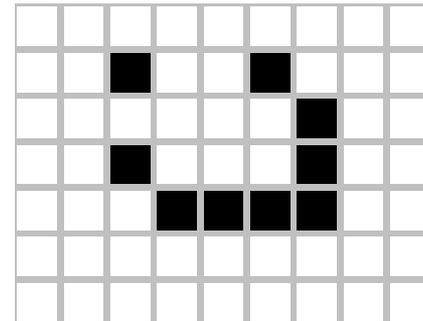
Conway's Game of Life [1970]



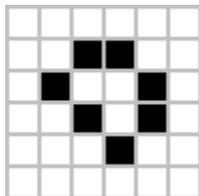
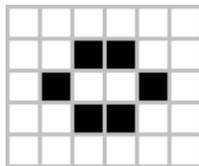
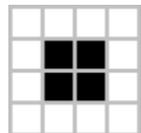
Pulsar



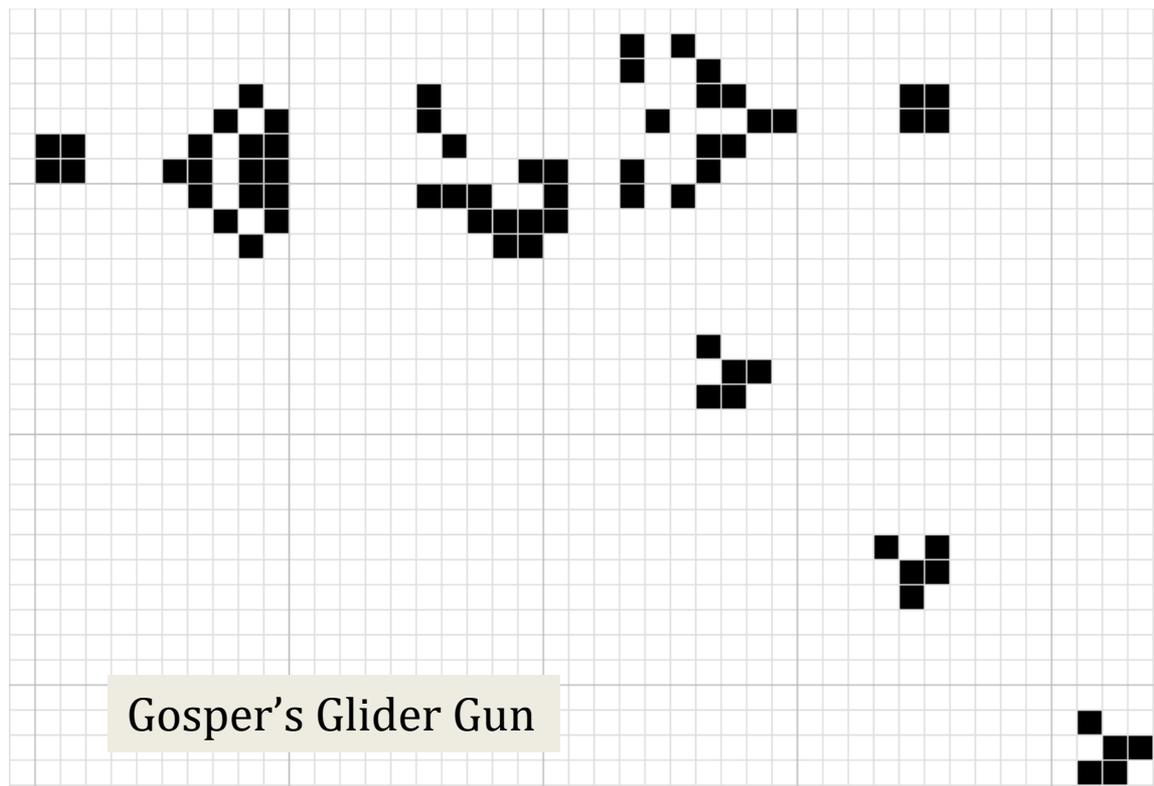
Glider



Spaceship



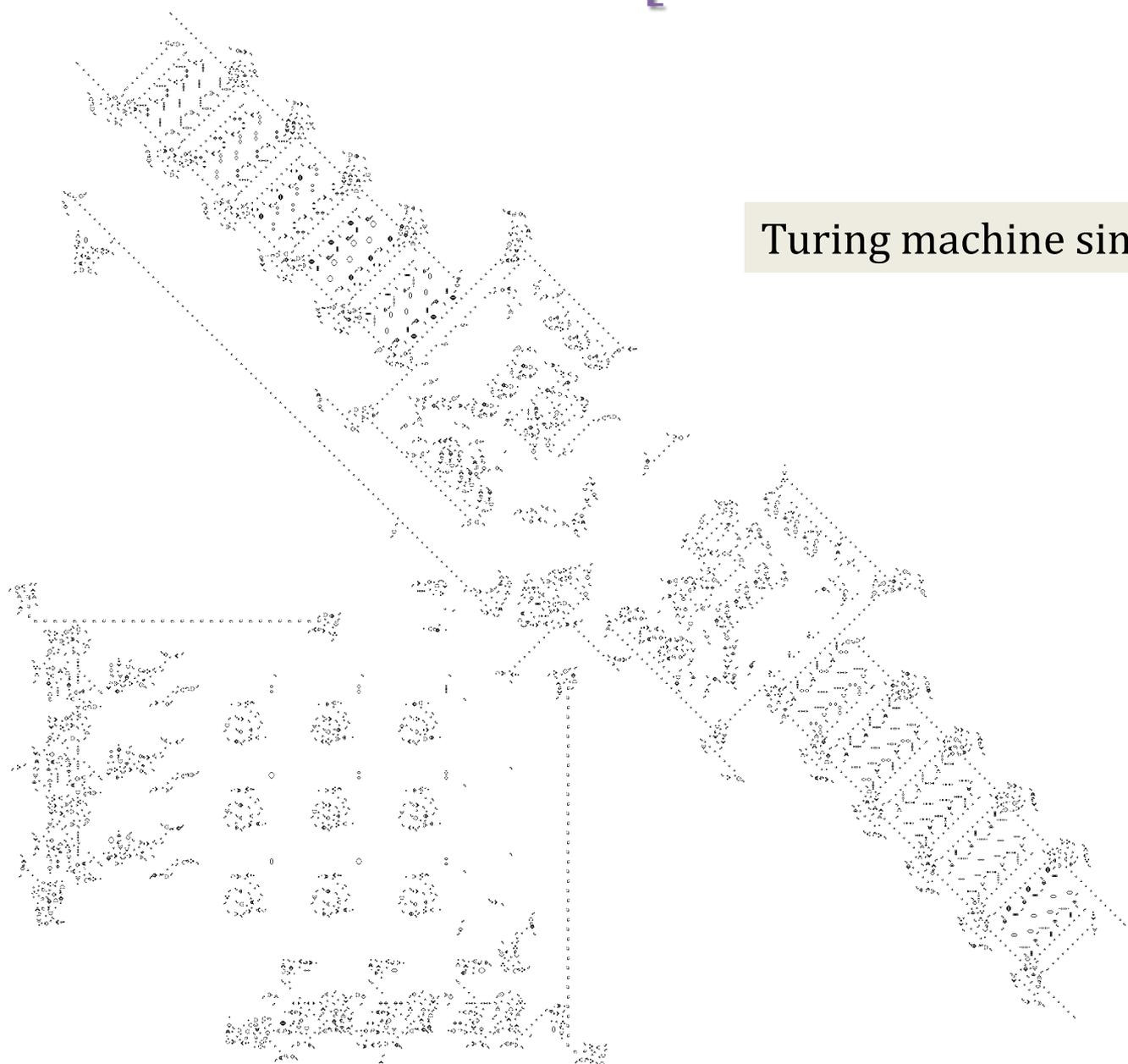
Still Life



Gosper's Glider Gun



Life is PSPACE-hard [Paul Rendell 2000]

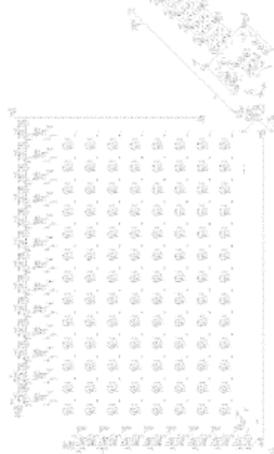


Turing machine simulation

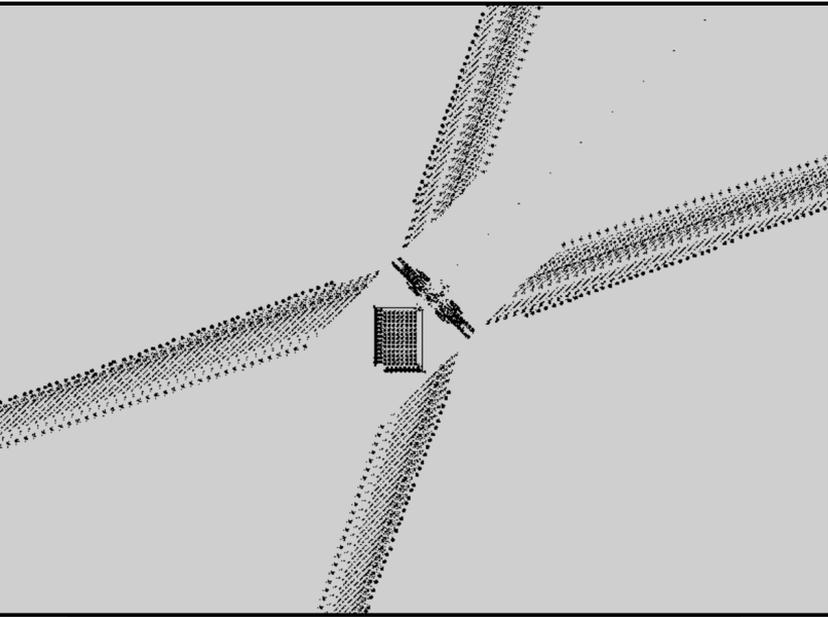


Life is PSPACE-hard [Paul Rendell 2000]

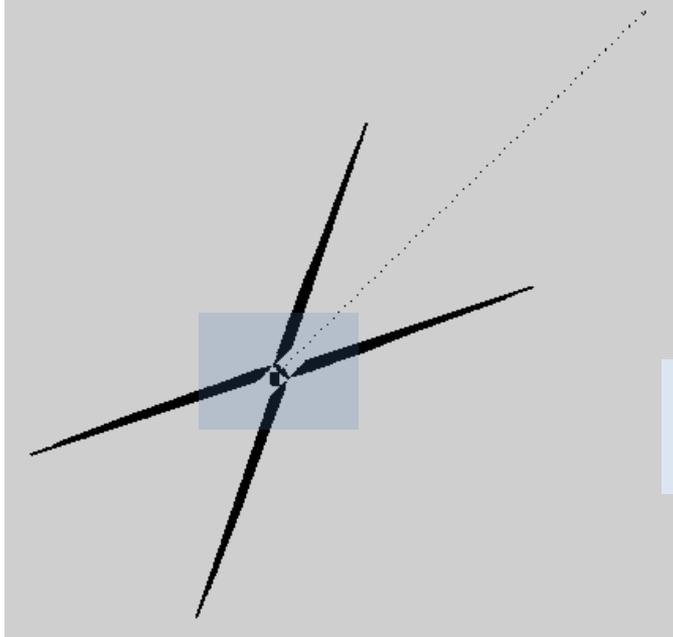
universal Turing machine simulation



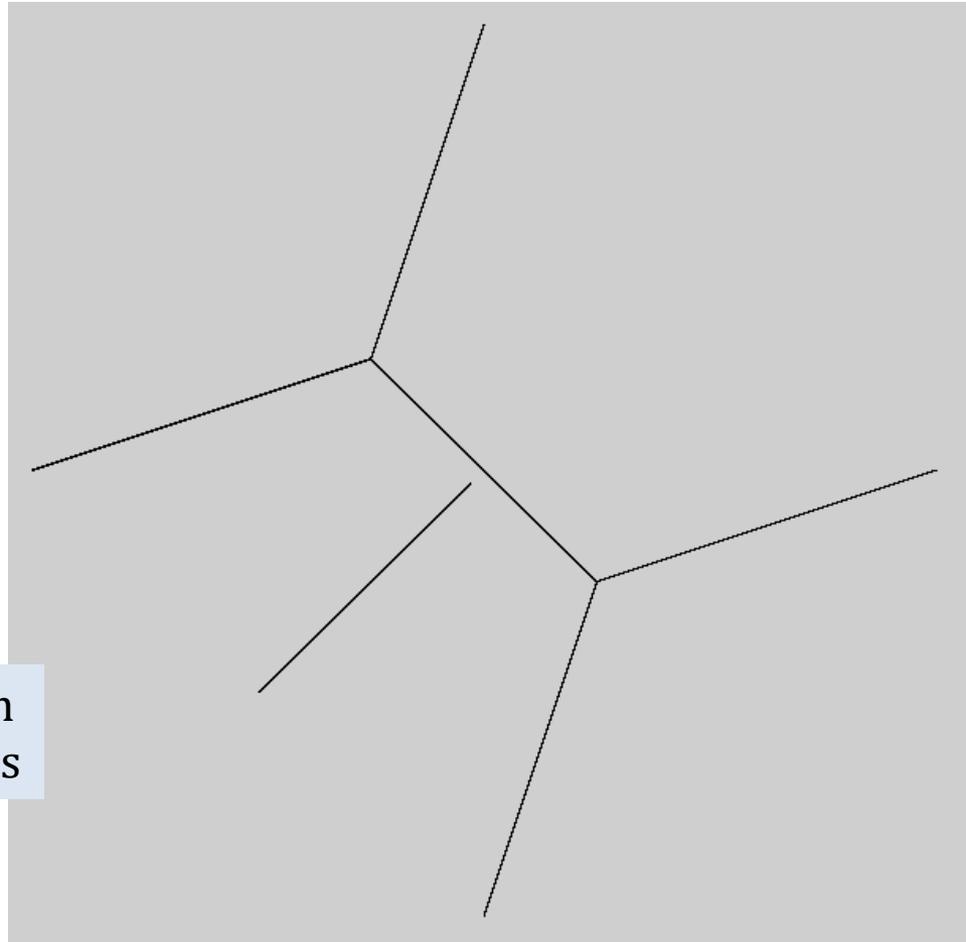
Life is Undecidable [Paul Rendell 2001]



universal Turing machine simulation



149 million generations



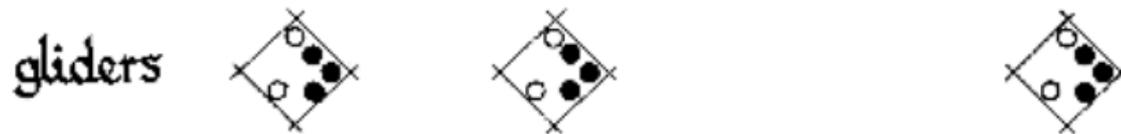


Life is Undecidable

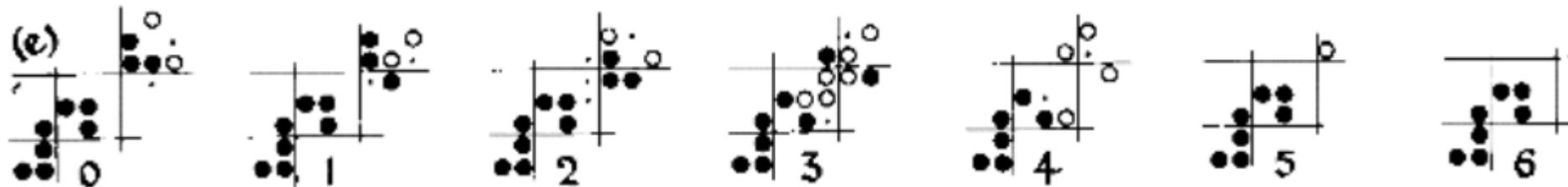
[Berlekamp, Conway, Guy 1982]



bits 1 1 0 1 0



wire

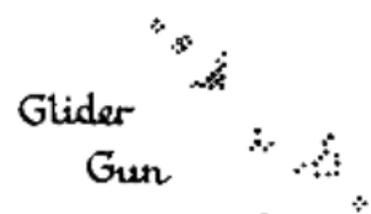


terminator



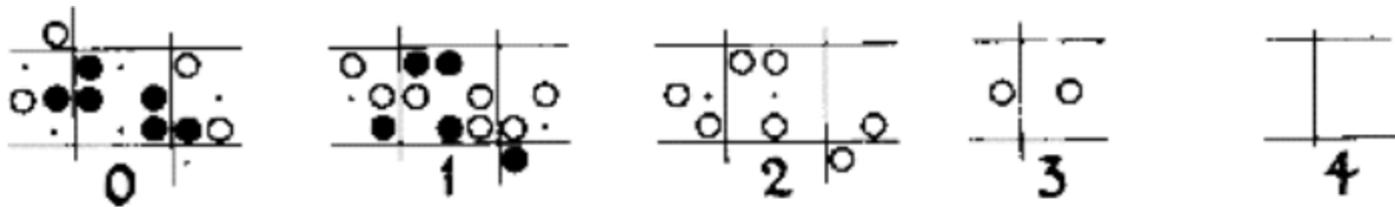
Life is Undecidable

[Berlekamp, Conway, Guy 1982]



turn

... 0 1 1 0 1 1 0 1 1 0 1
 Input



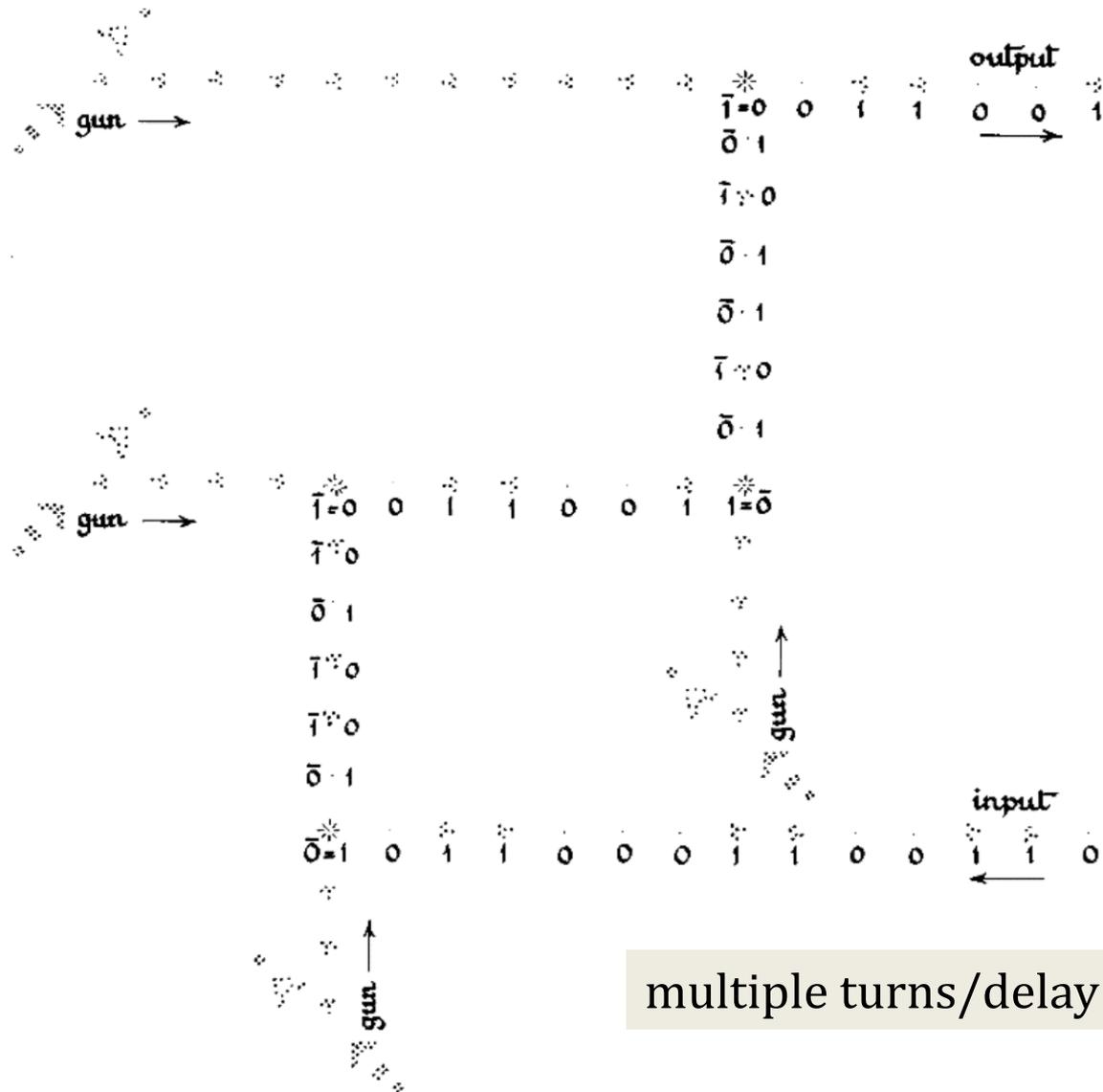
Output

1
 0
 0
 1
 0
 0
 1
 0
 0
 ⋮



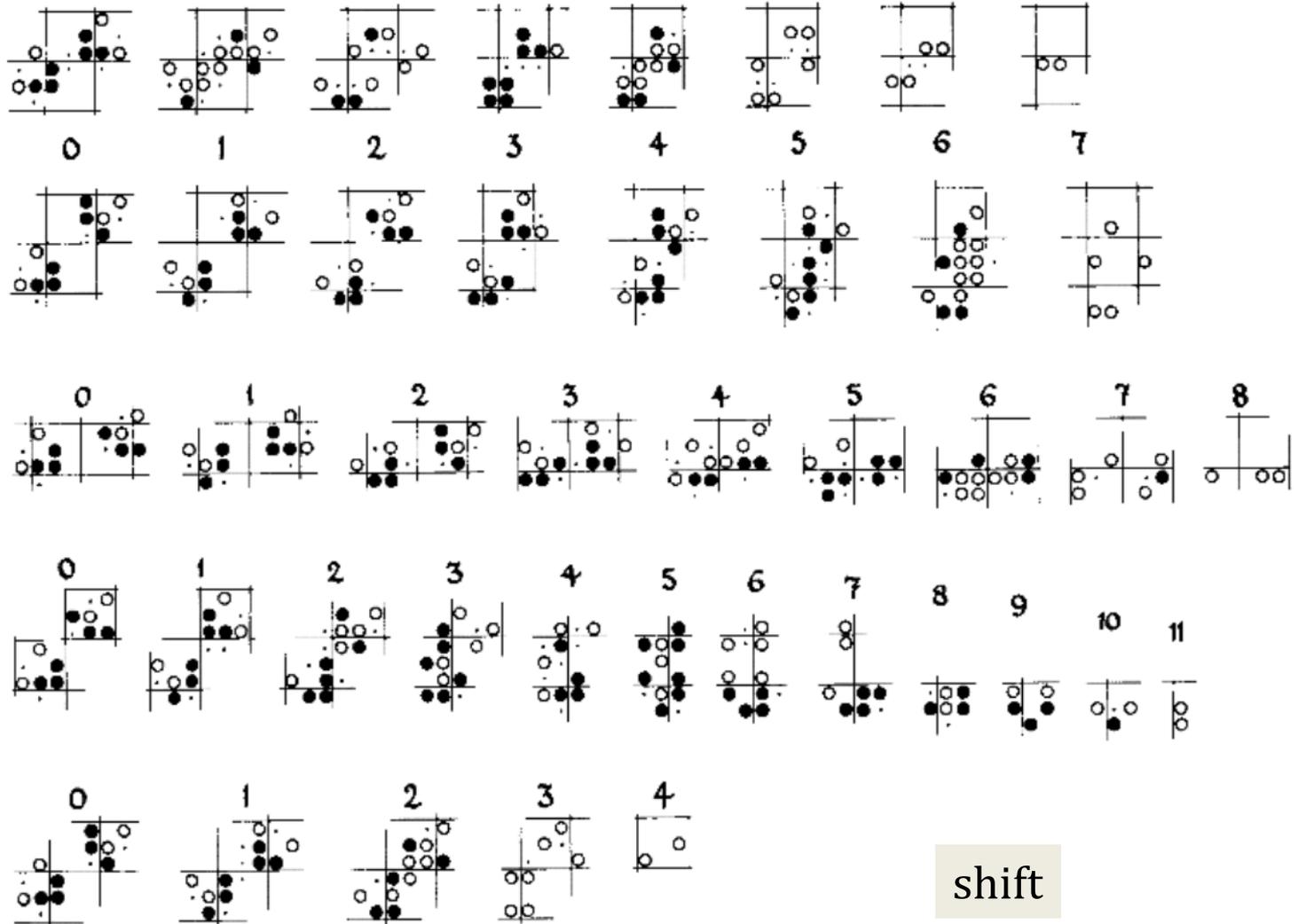
Life is Undecidable

[Berlekamp, Conway, Guy 1982]



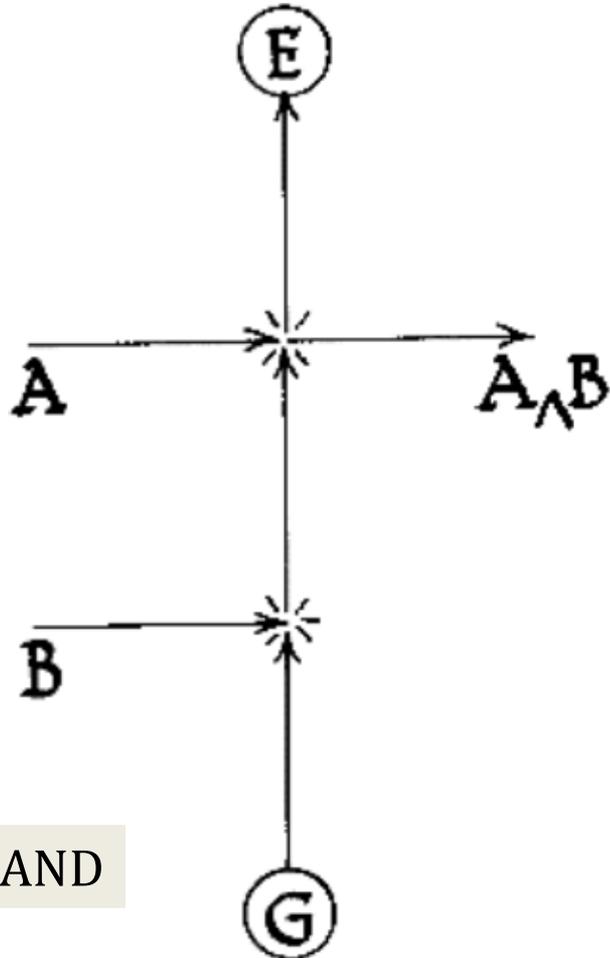
Life is Undecidable

[Berlekamp, Conway, Guy 1982]

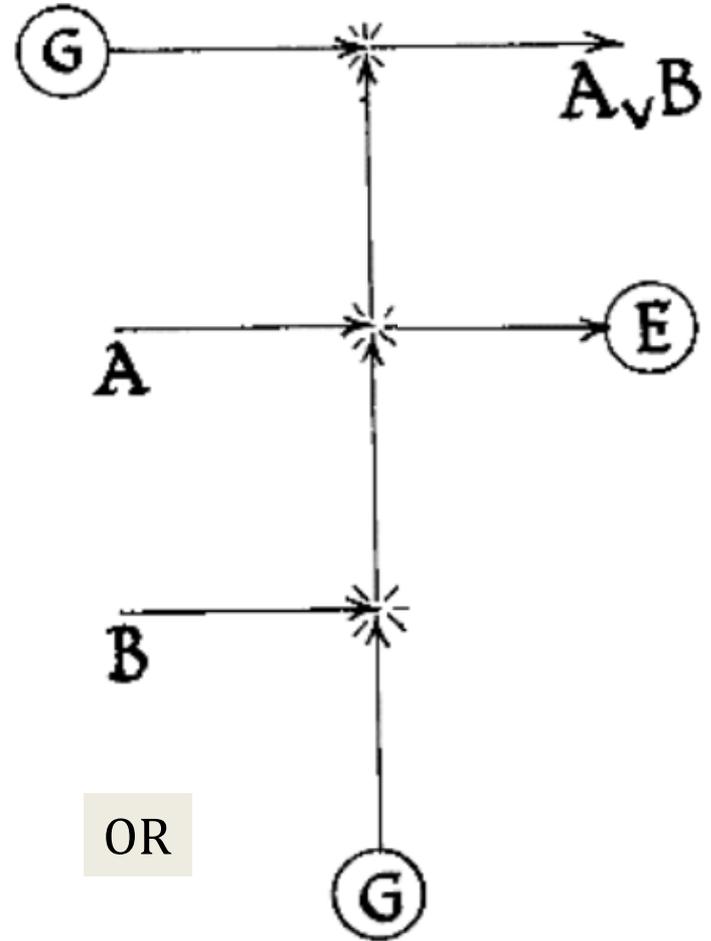


Life is Undecidable

[Berlekamp, Conway, Guy 1982]



AND

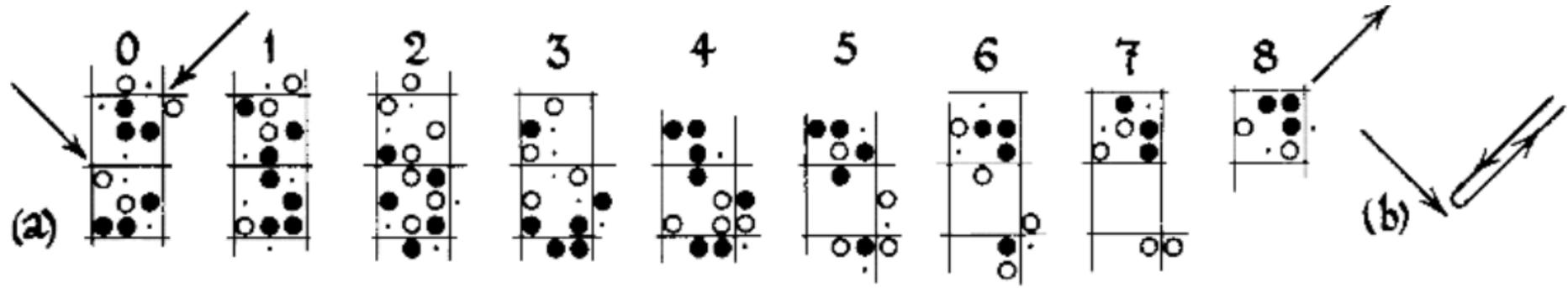


OR



Life is Undecidable

[Berlekamp, Conway, Guy 1982]

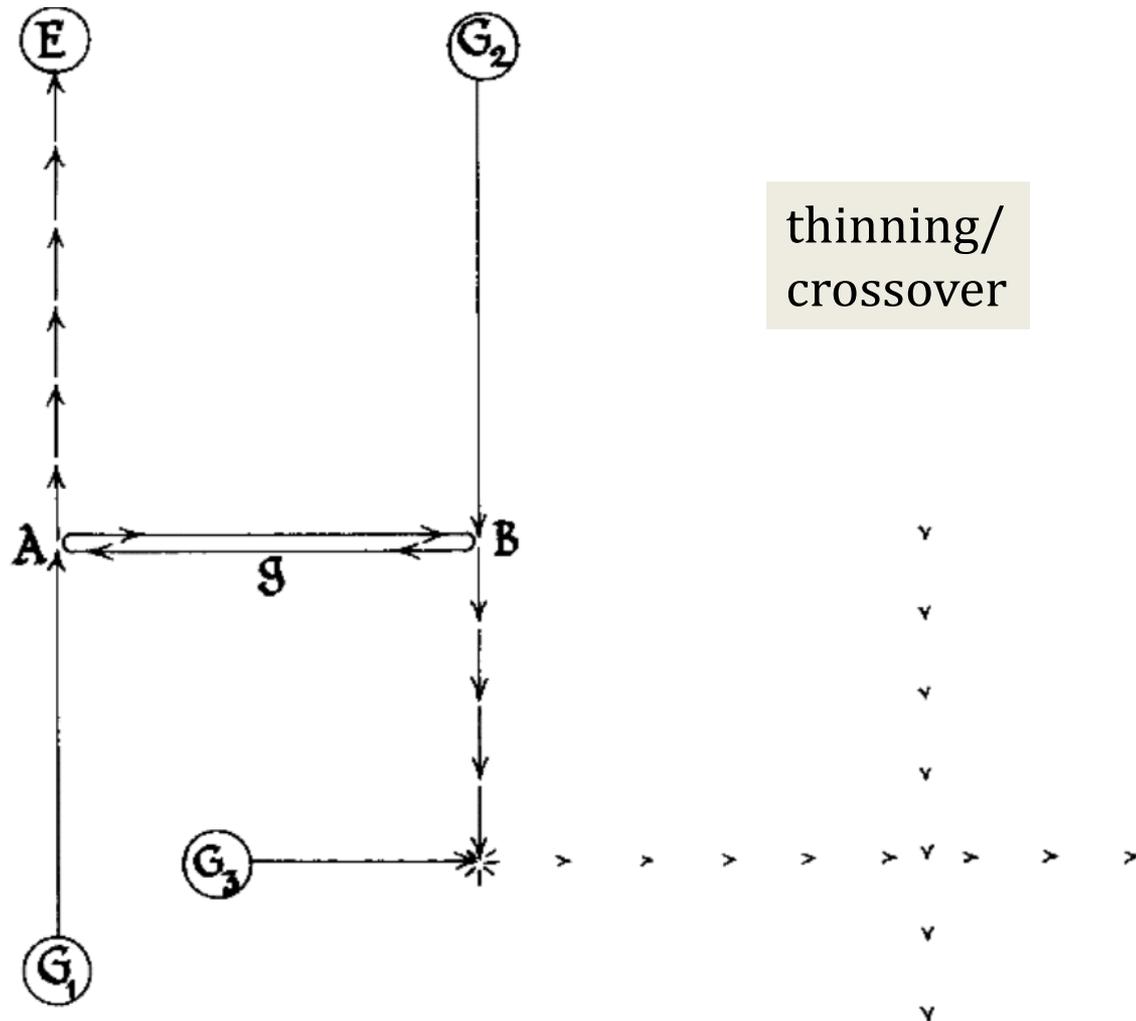


kick back



Life is Undecidable

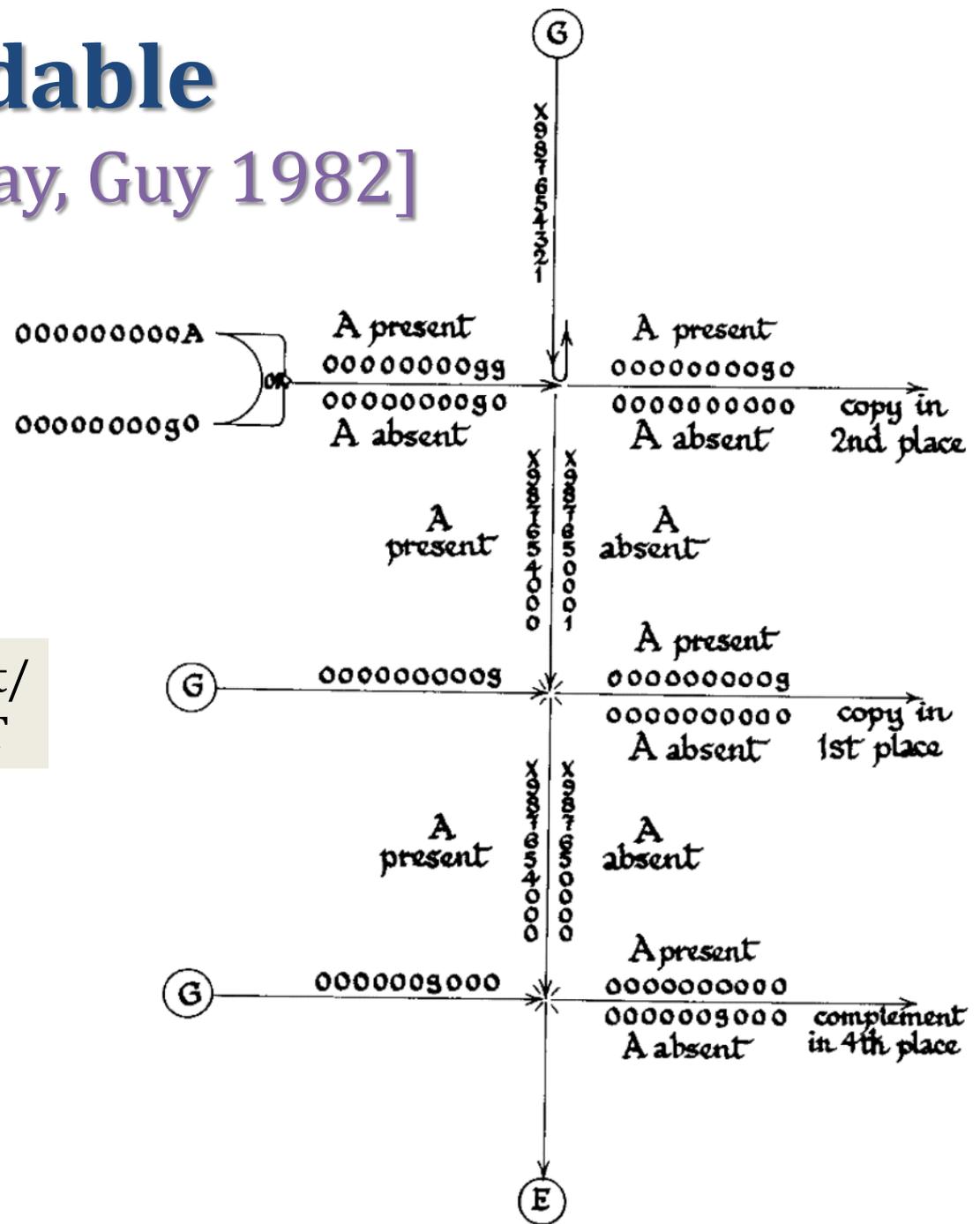
[Berlekamp, Conway, Guy 1982]



Life is Undecidable

[Berlekamp, Conway, Guy 1982]

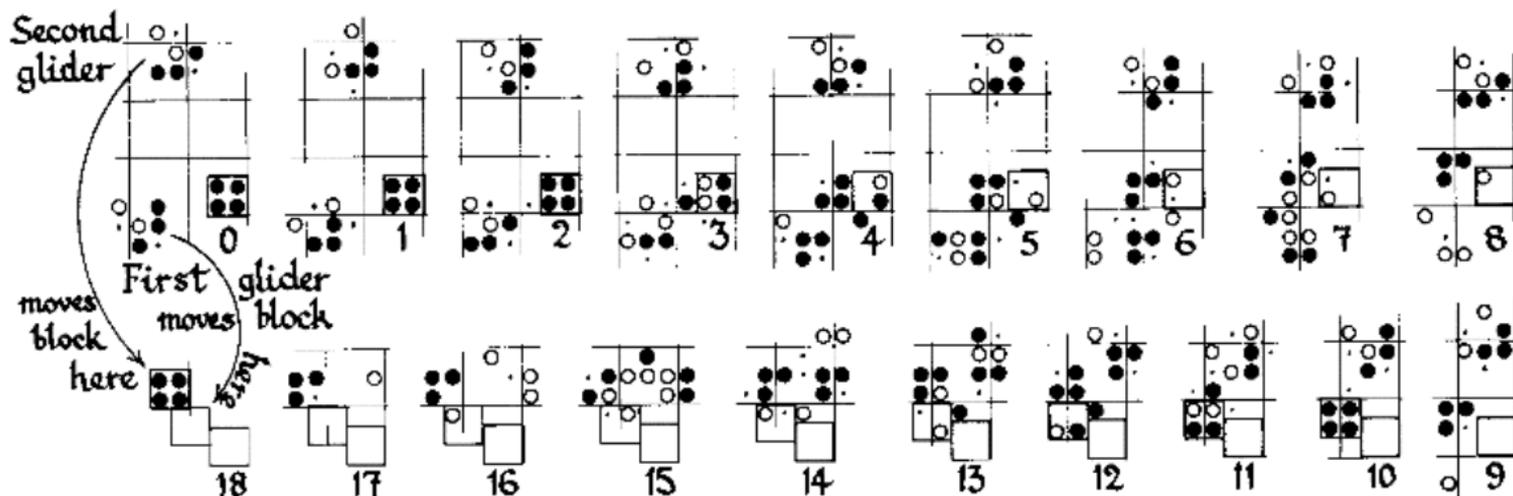
split/
NOT



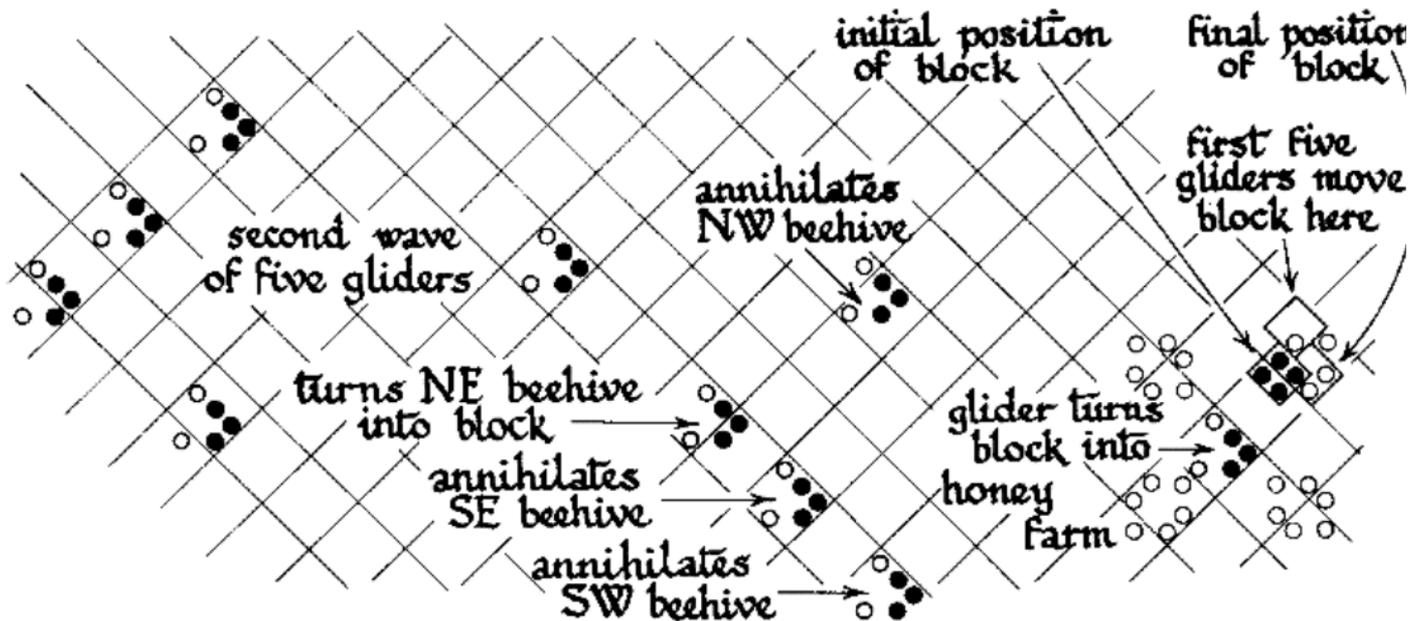
Life is Undecidable

[Berlekamp, Conway, Guy 1982]

pull block
back 3



push block
forward 1

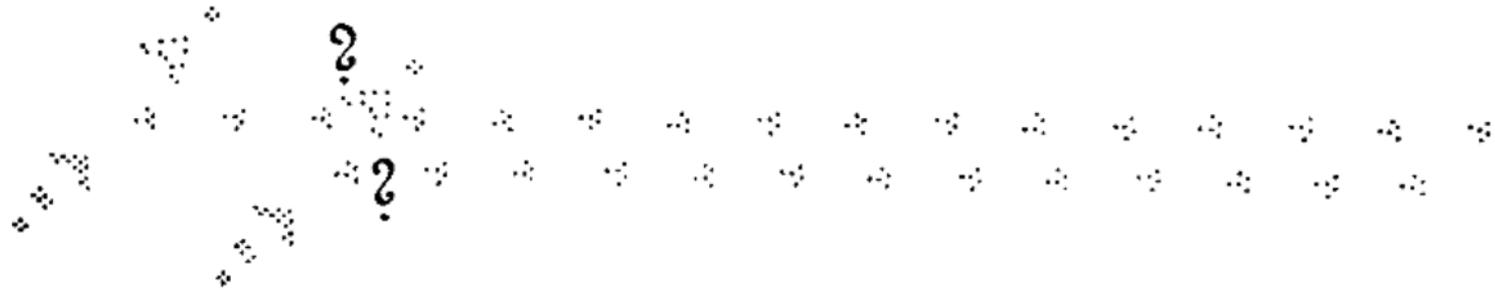




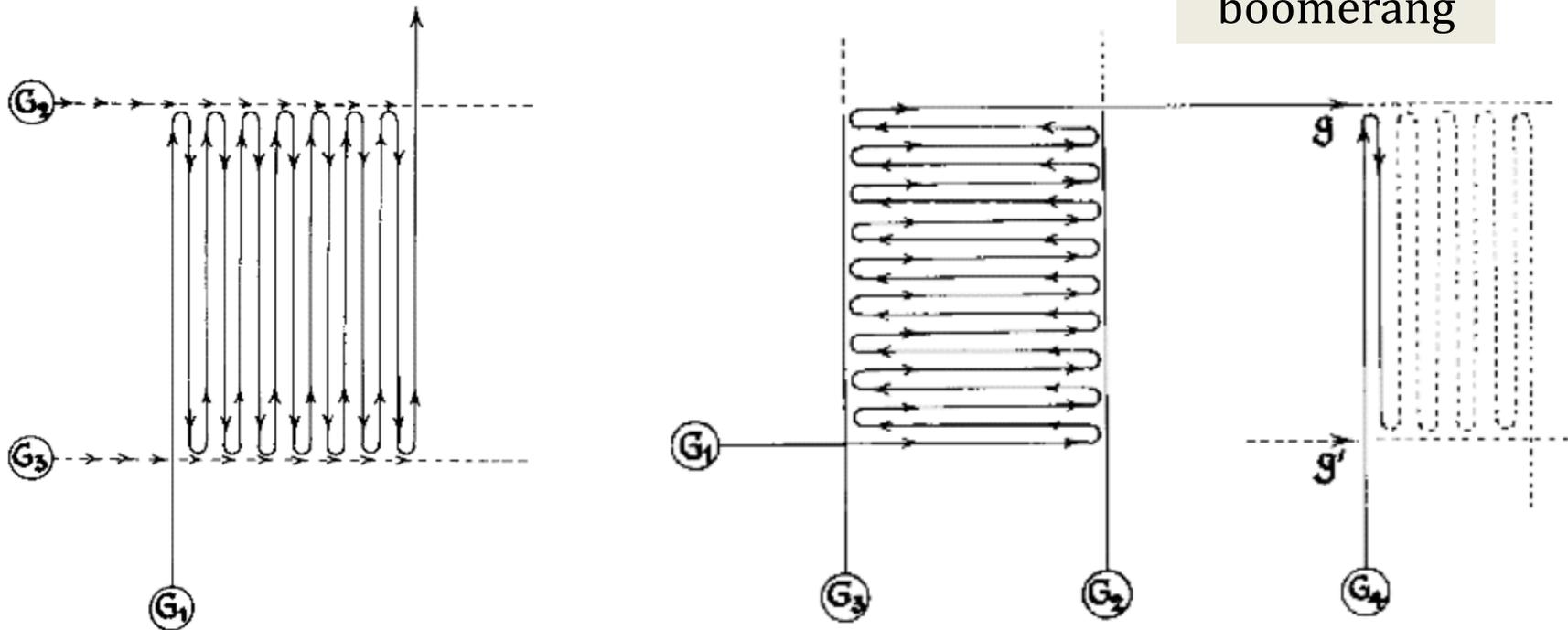
Life is Undecidable

[Berlekamp, Conway, Guy 1982]

precise
glider
positioning



boomerang

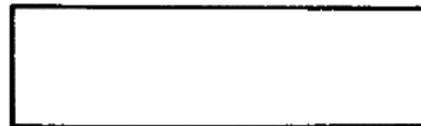


Life is Undecidable

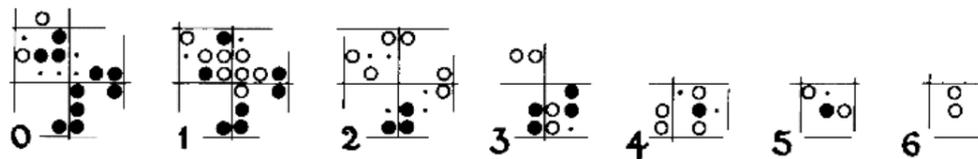
[Berlekamp, Conway, Guy 1982]



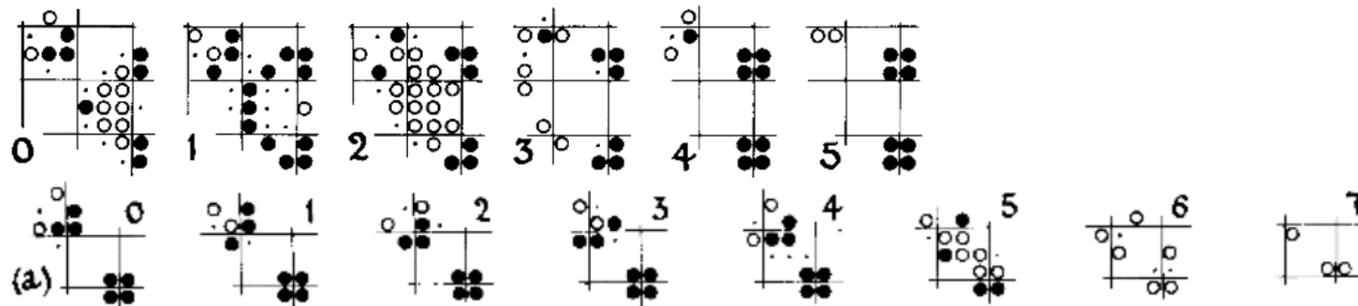
self-destruction



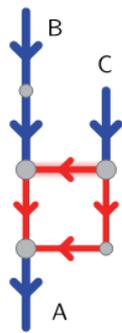
destroy terminator



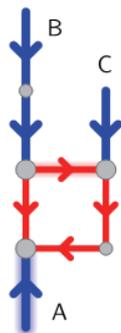
destroy gun



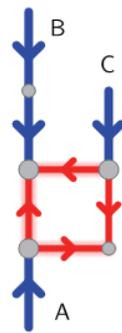
DCL Switch



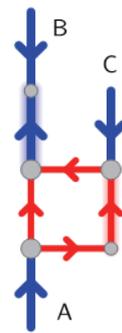
(a) $t = 0$



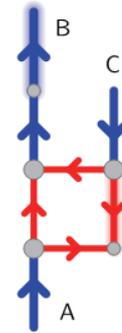
(b) $t = 1$



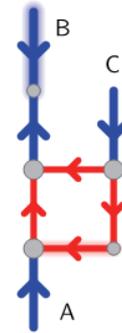
(c) $t = 2$



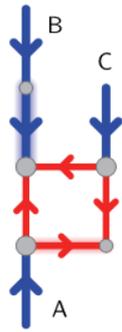
(d) $t = 3$



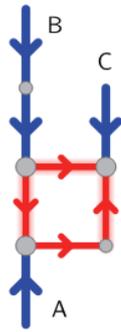
(e) $t = 4$



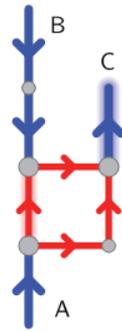
(f) $t = 5 + 4k$



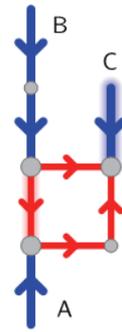
(g) $t = 6 + 4k$



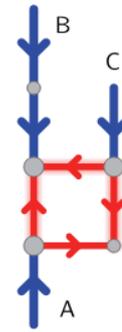
(h) $t = 7 + 4k$



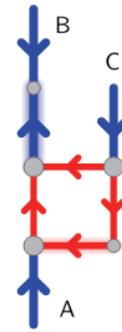
(i) $t = 8 + 4k$



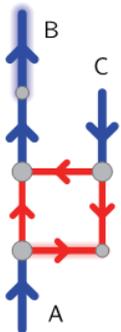
(j) $t = 9 + 4l$



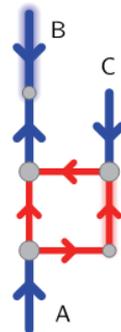
(k) $t = 10 + 4l$



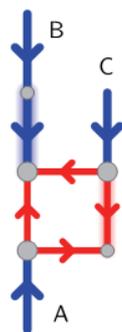
(l) $t = 11 + 4l$



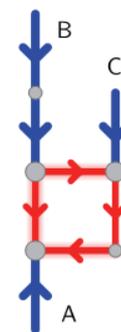
(m) $t = 12 + 4l$



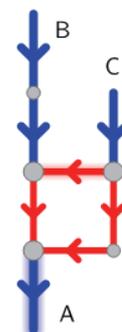
(n) $t = 13 + 4m$



(o) $t = 14 + 4m$



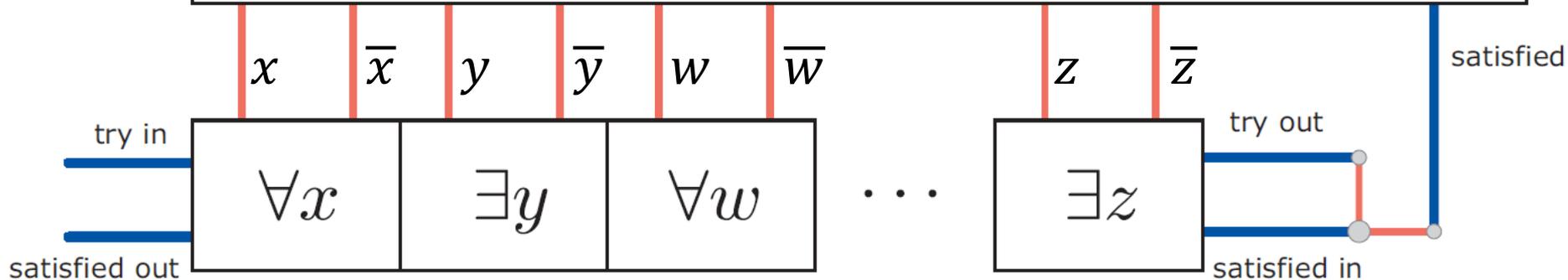
(p) $t = 15 + 4m$



(q) $t = 16 + 4m$

Reduction from Q-CNF-SAT

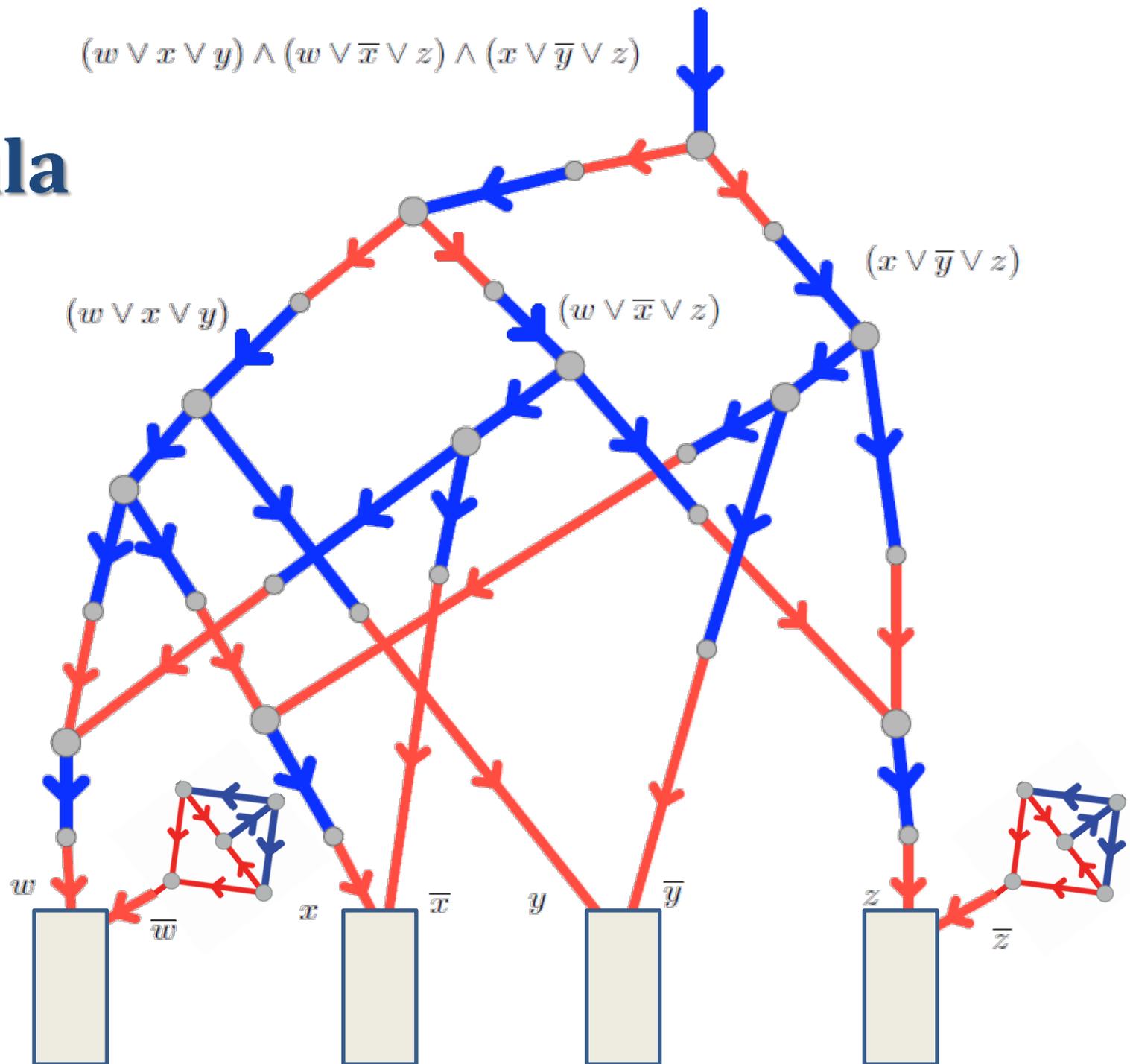
$$\forall x \exists y \forall w \dots \exists z [(x \vee y) \wedge \dots \wedge (\bar{z} \vee x \vee \bar{w})]$$





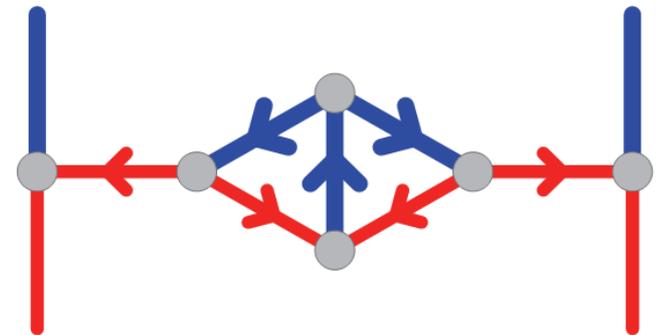
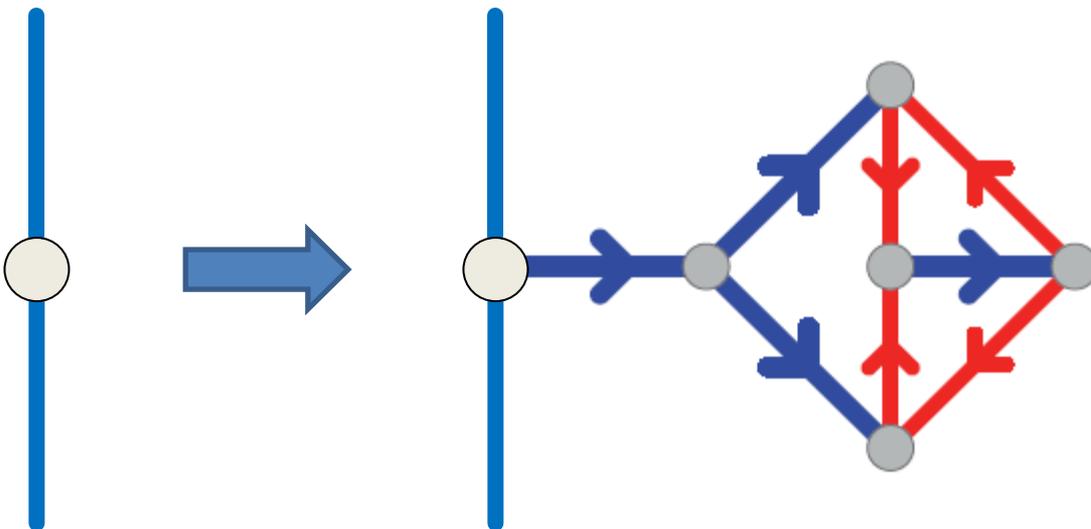
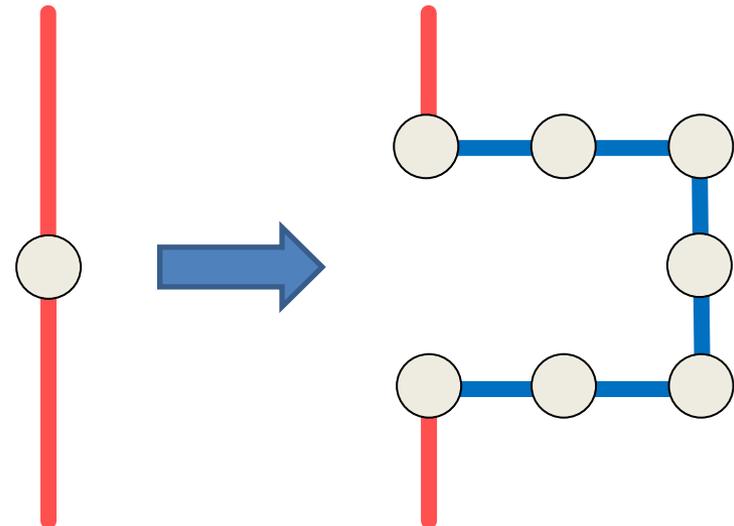
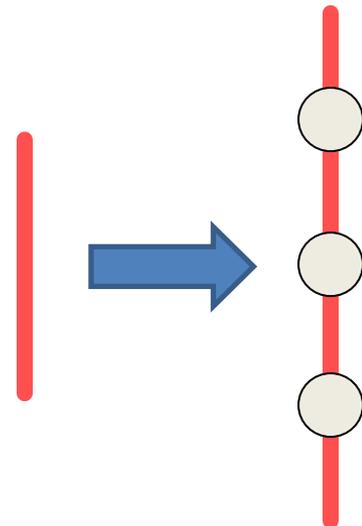
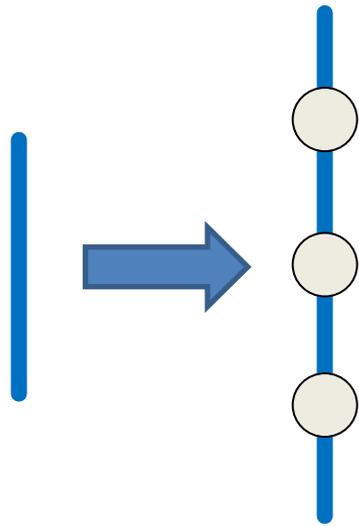
CNF Formula

$$(w \vee x \vee y) \wedge (w \vee \bar{x} \vee z) \wedge (x \vee \bar{y} \vee z)$$

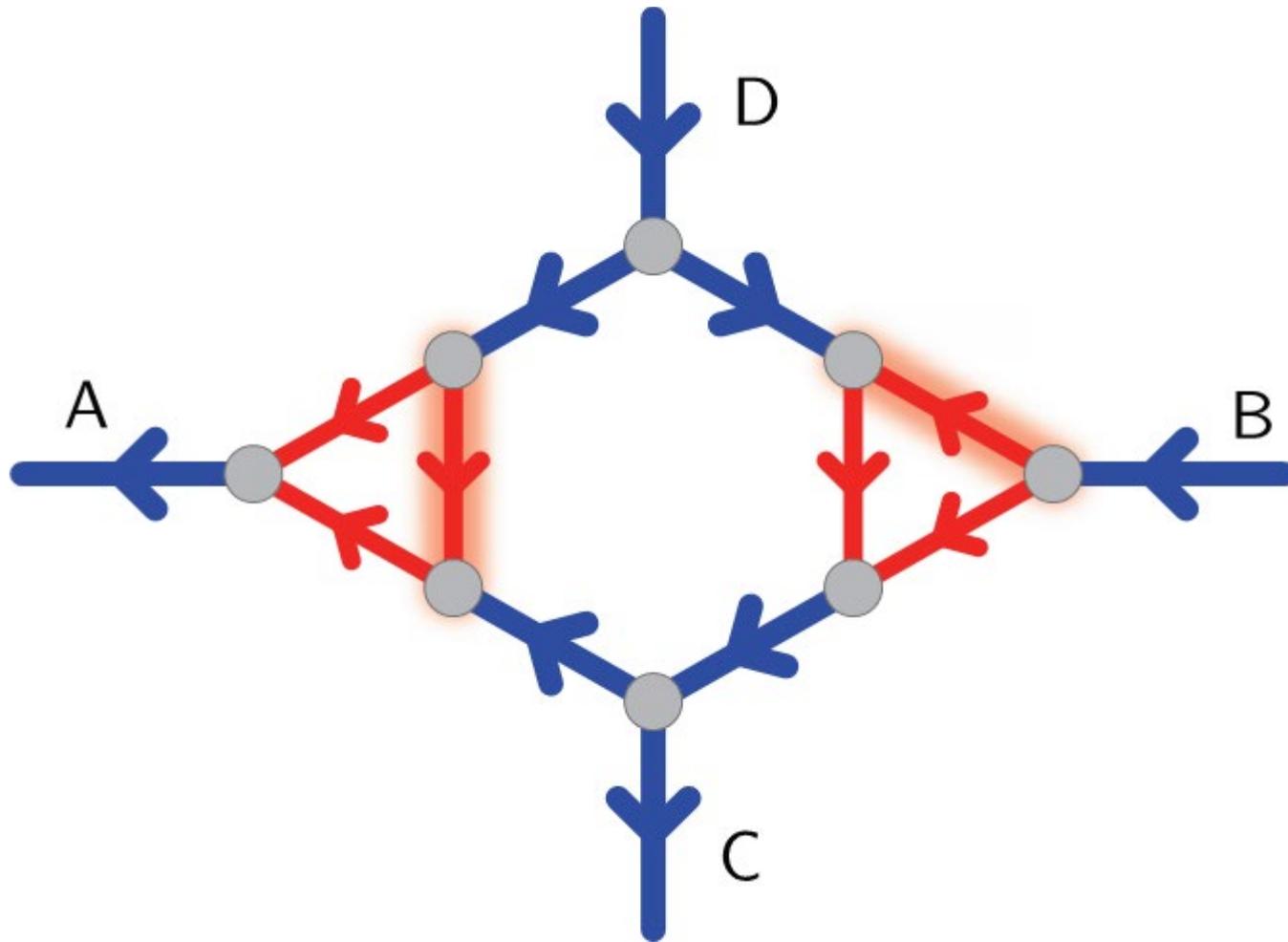




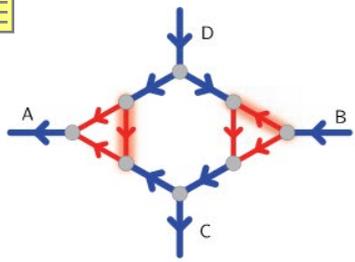
Removing Degree-2 Vertices



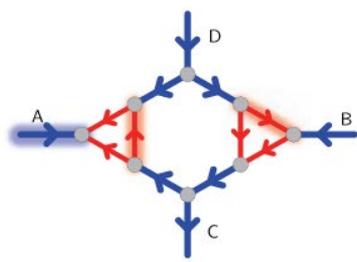
Planar DCL



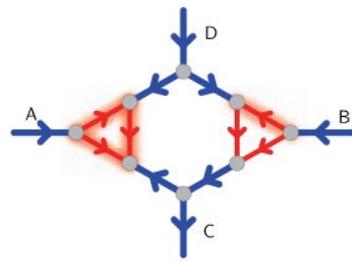
Planar DCL



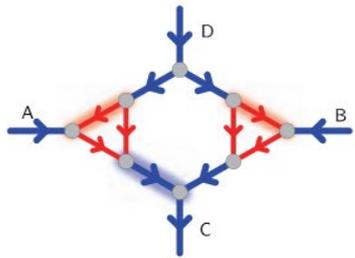
(a) $t = 0$, signal about to arrive at A



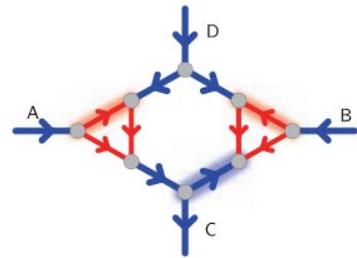
(b) $t = 1$



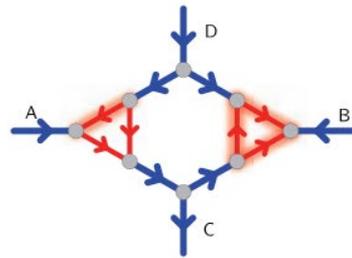
(c) $t = 2$



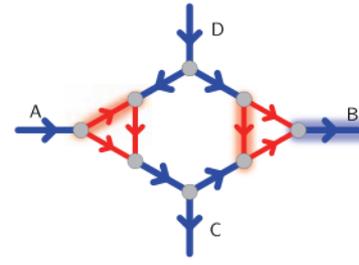
(d) $t = 3$



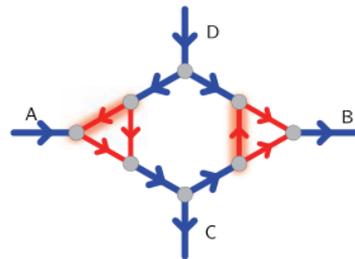
(e) $t = 4$



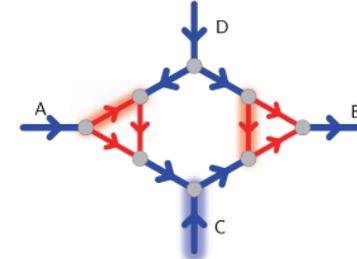
(f) $t = 5$



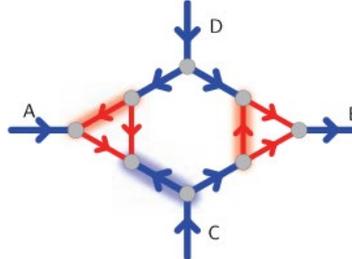
(g) $t = 6$



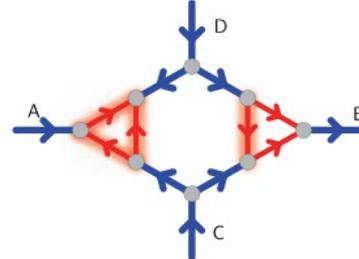
(h) $t = 9 + 4k$, signal about to arrive at C



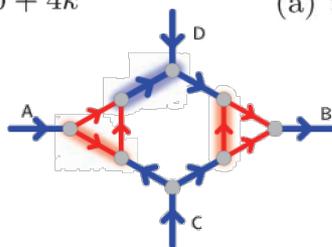
(i) $t = 10 + 4k$



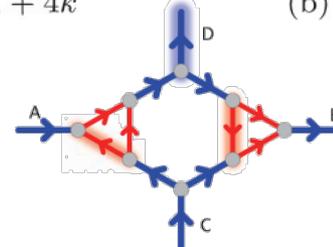
(a) $t = 11 + 4k$



(b) $t = 12 + 4k$



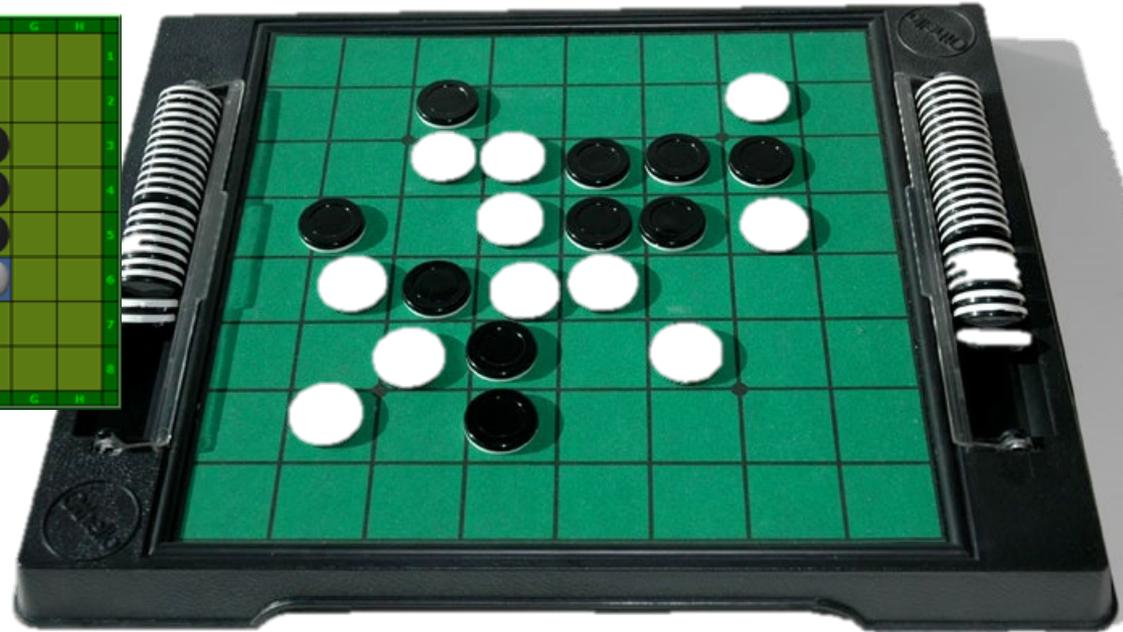
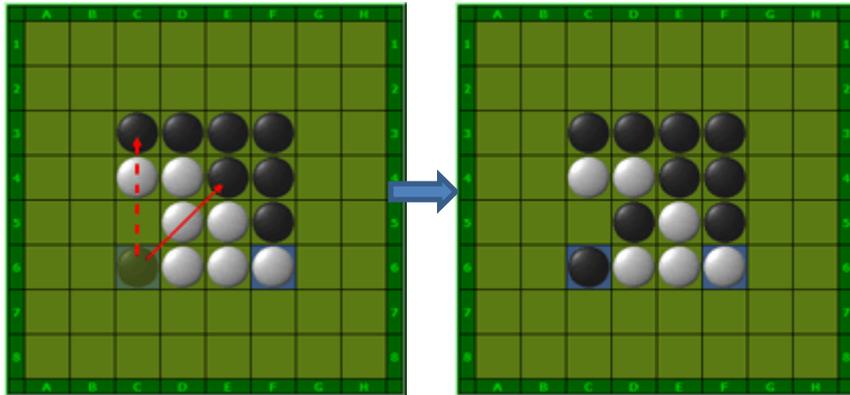
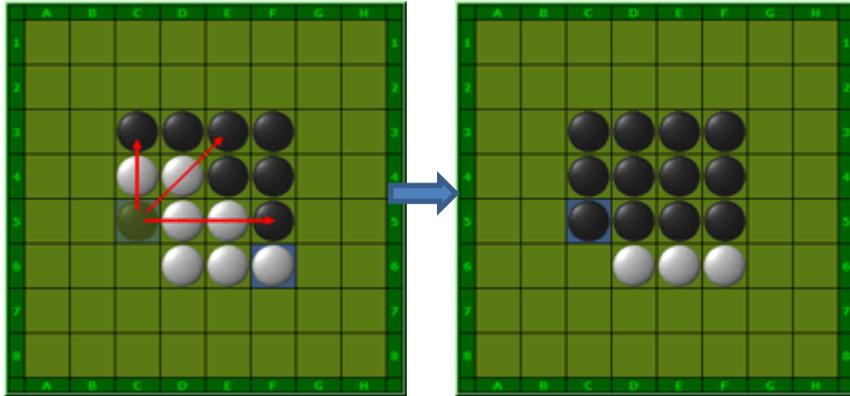
(c) $t = 13 + 4k$



(d) $t = 14 + 4k$

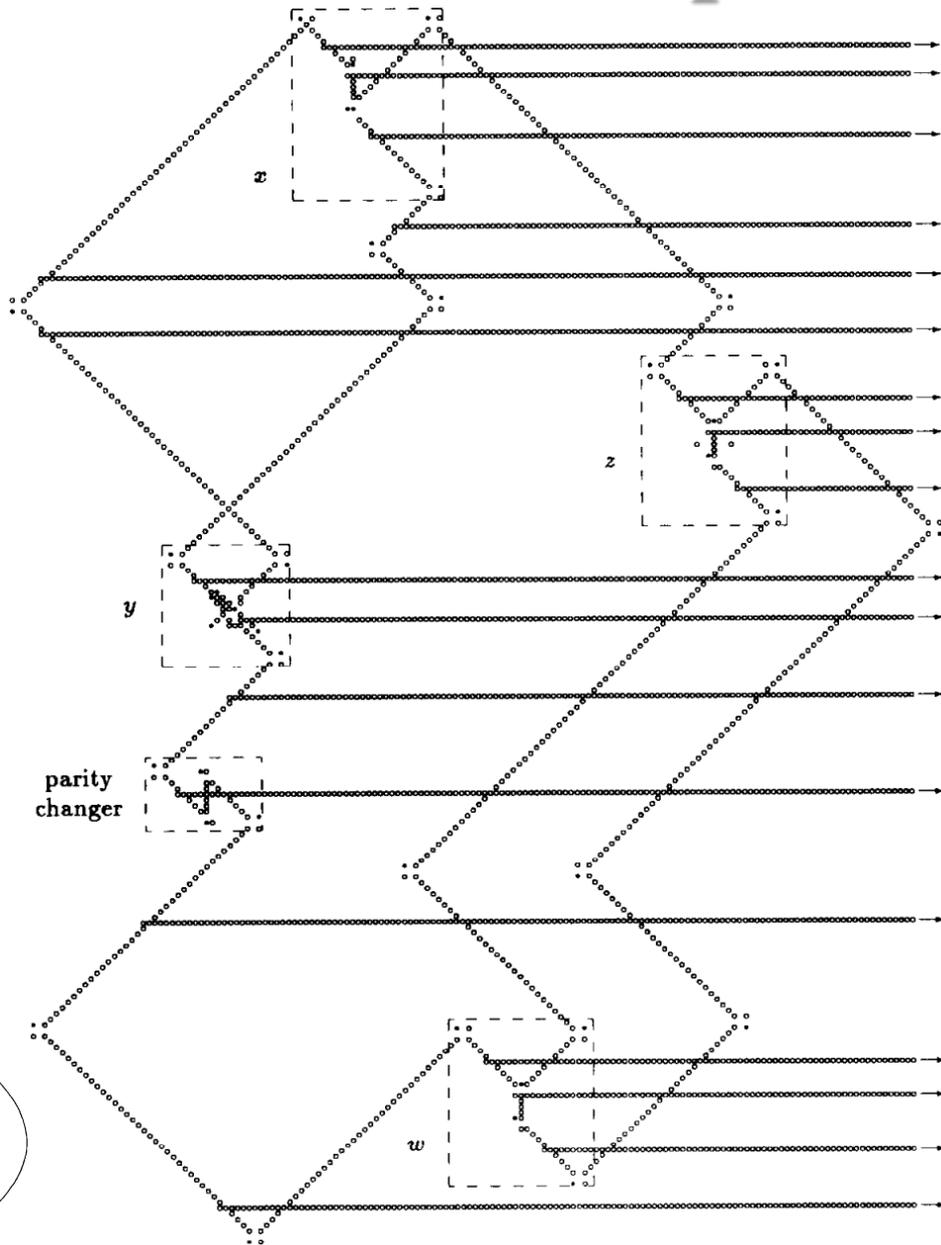
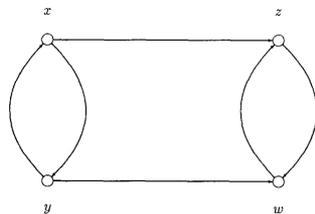
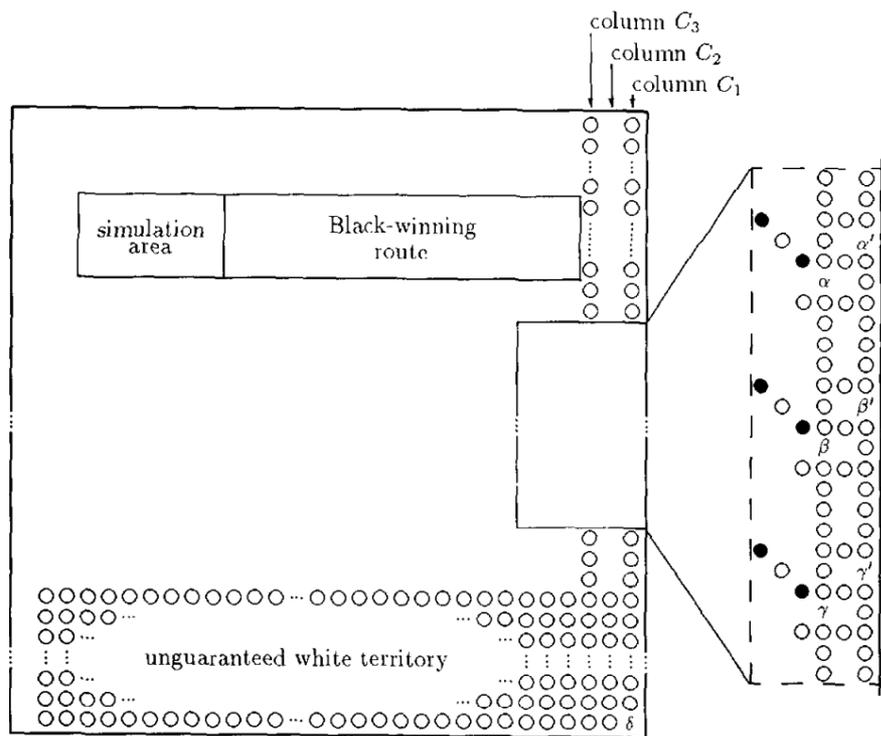
Reversi/Othello

[<1883]



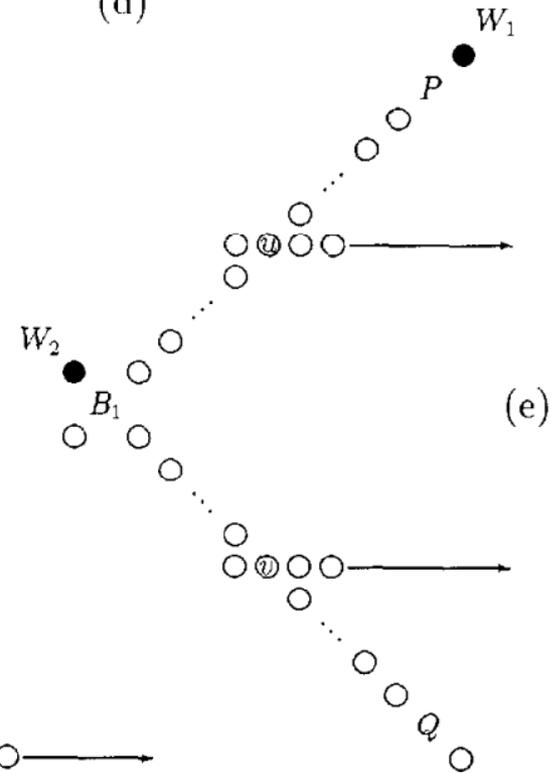
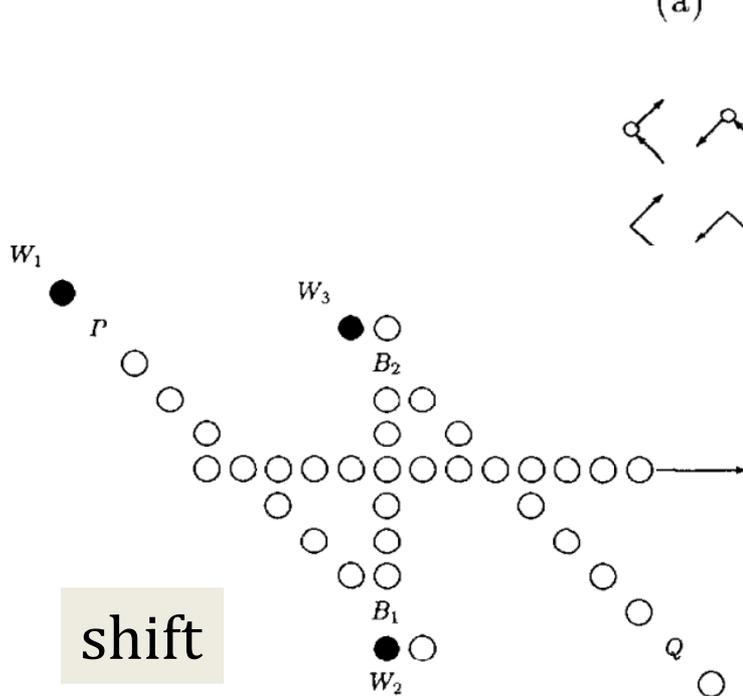
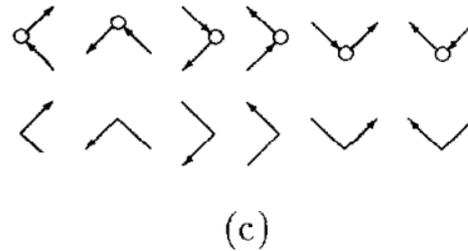
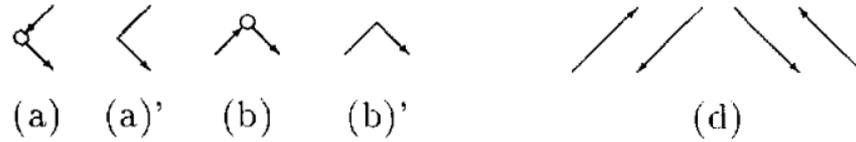
Reversi/Othello is PSPACE-complete

[Iwata & Kasai 1994]



Reversi/Othello is PSPACE-complete

[Iwata & Kasai 1994]



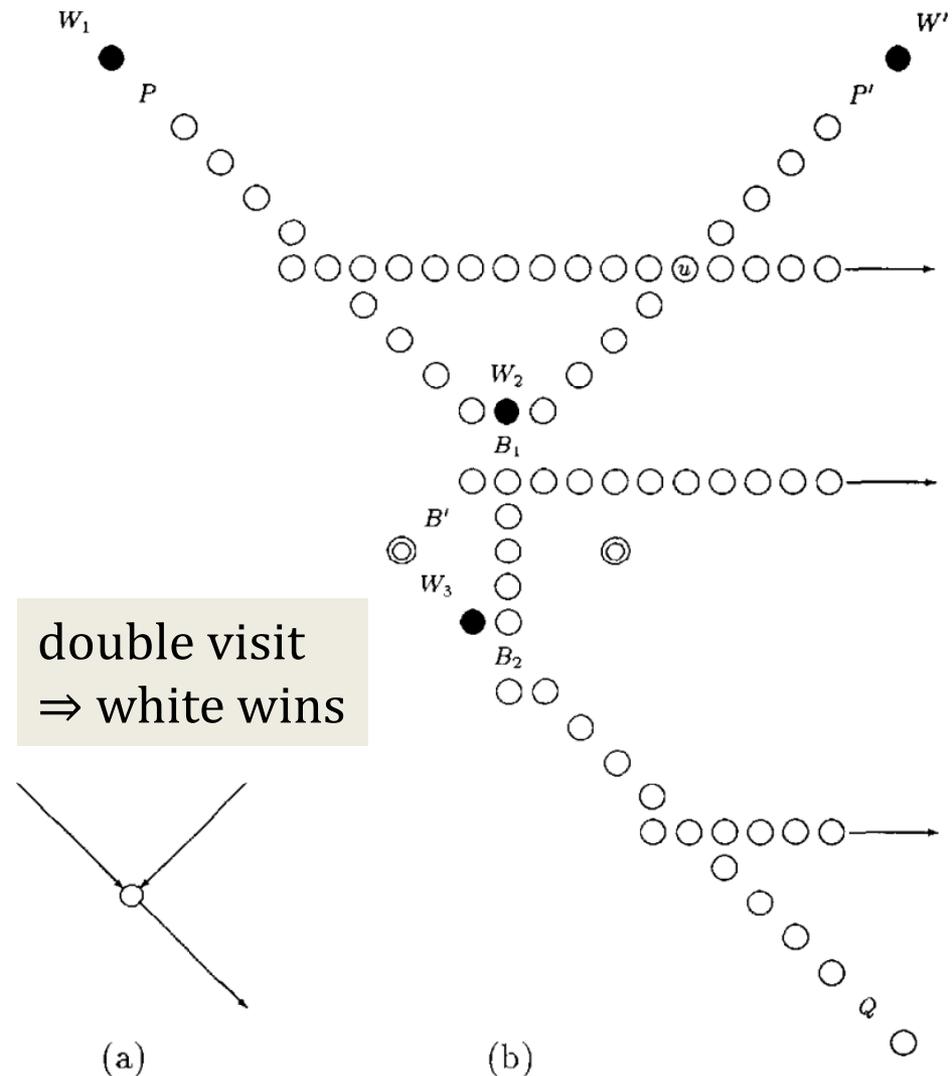
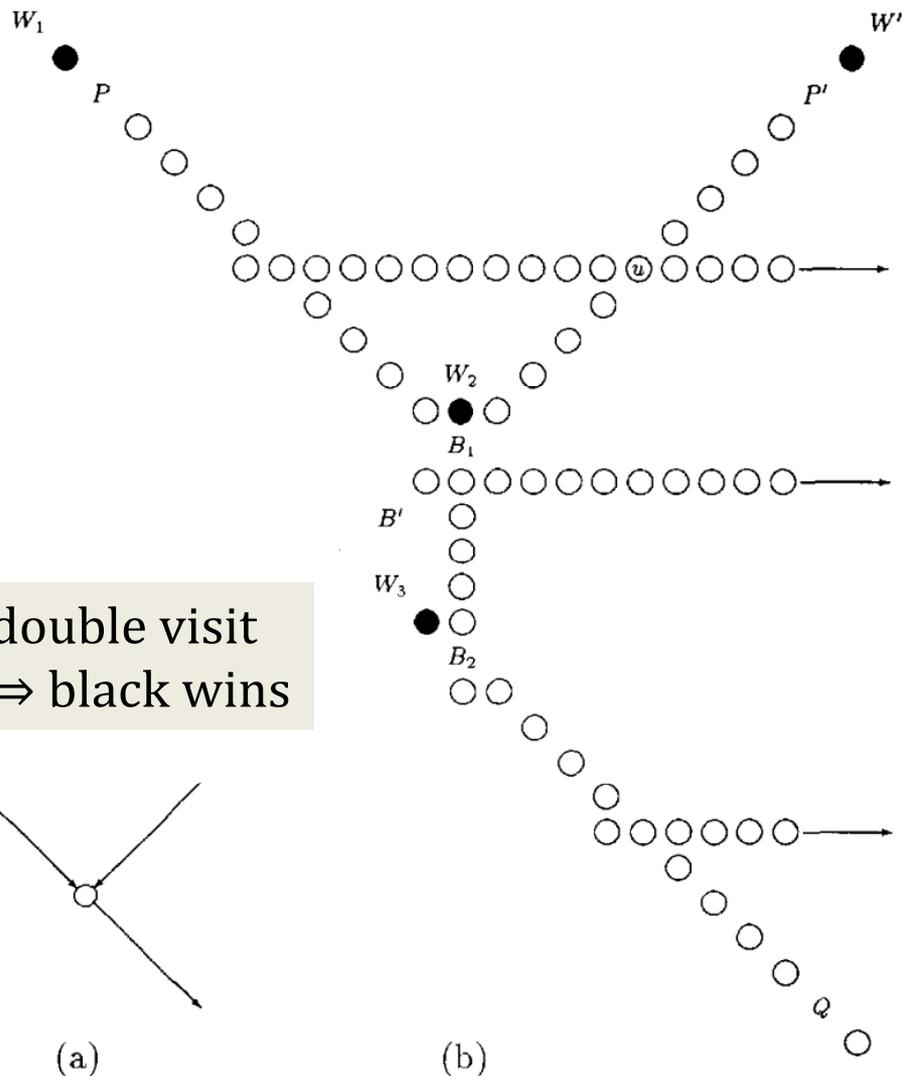
shift

turn/
degree 2



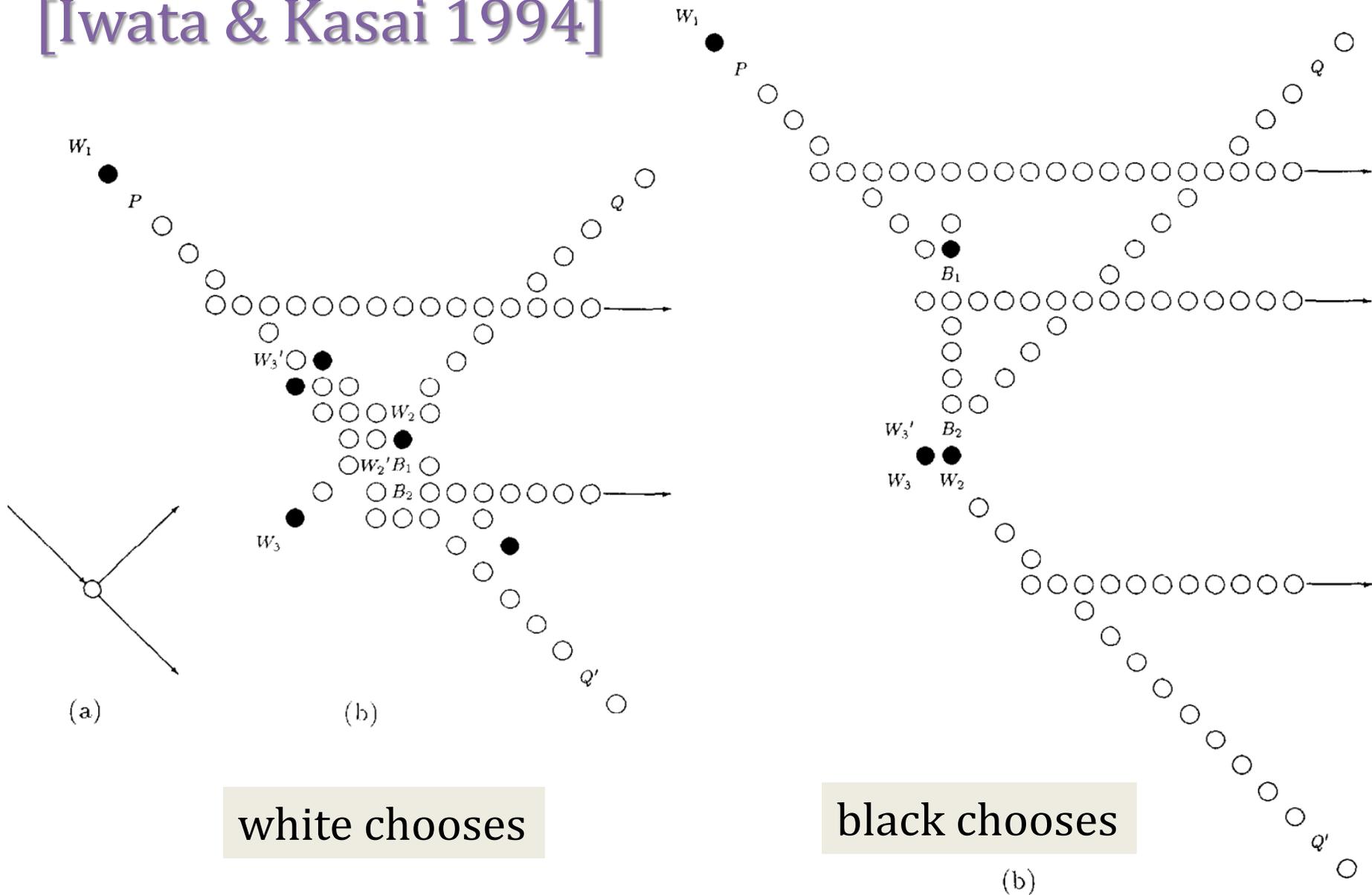
Reversi/Othello is PSPACE-complete

[Iwata & Kasai 1994]



Reversi/Othello is PSPACE-complete

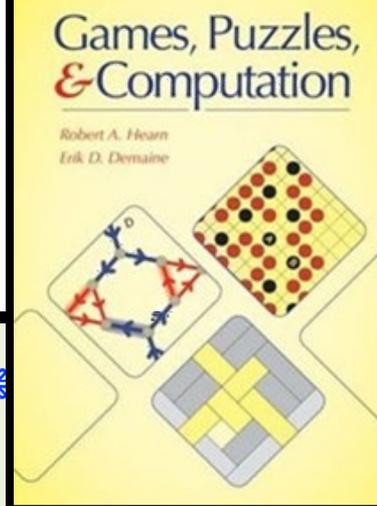
[Iwata & Kasai 1994]



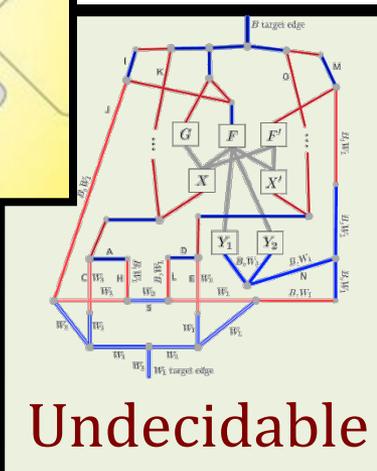
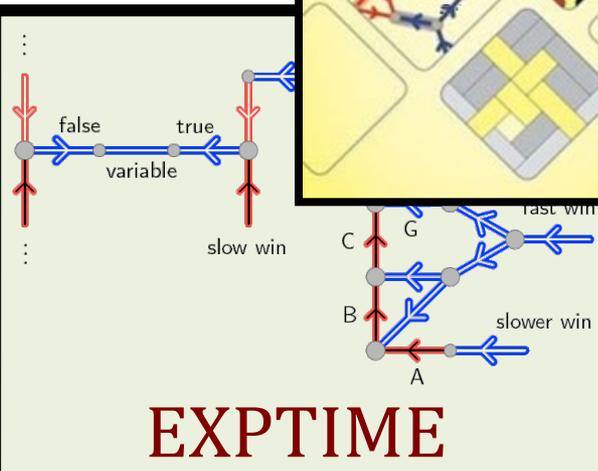
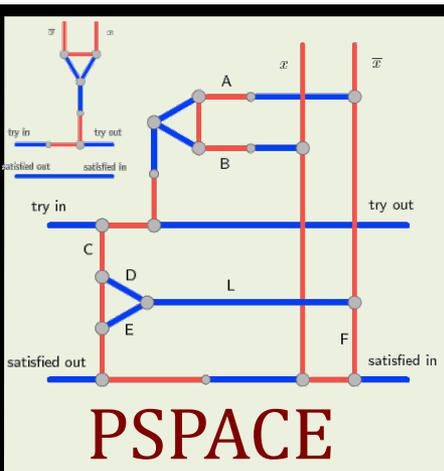
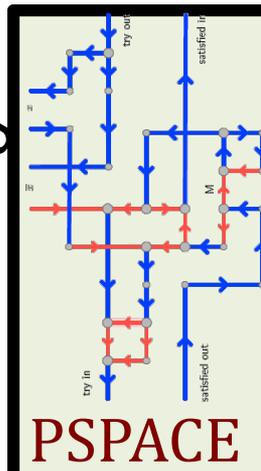


Constraint Logic

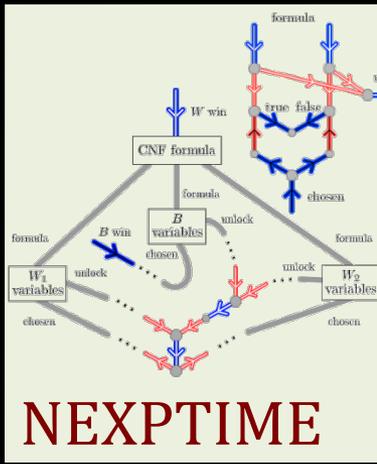
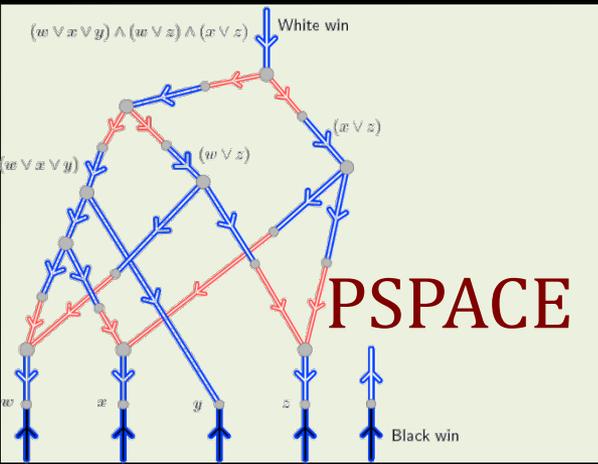
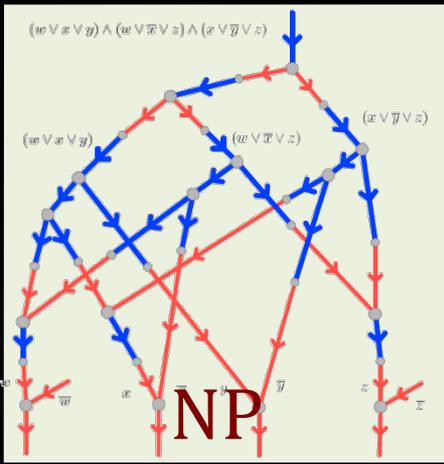
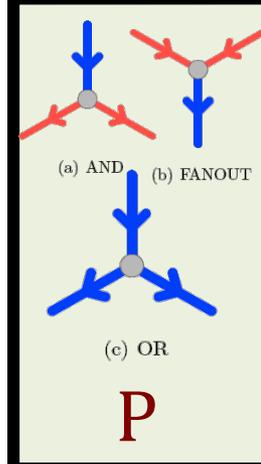
[Hearn & Demaine 2009]



unbounded



bounded



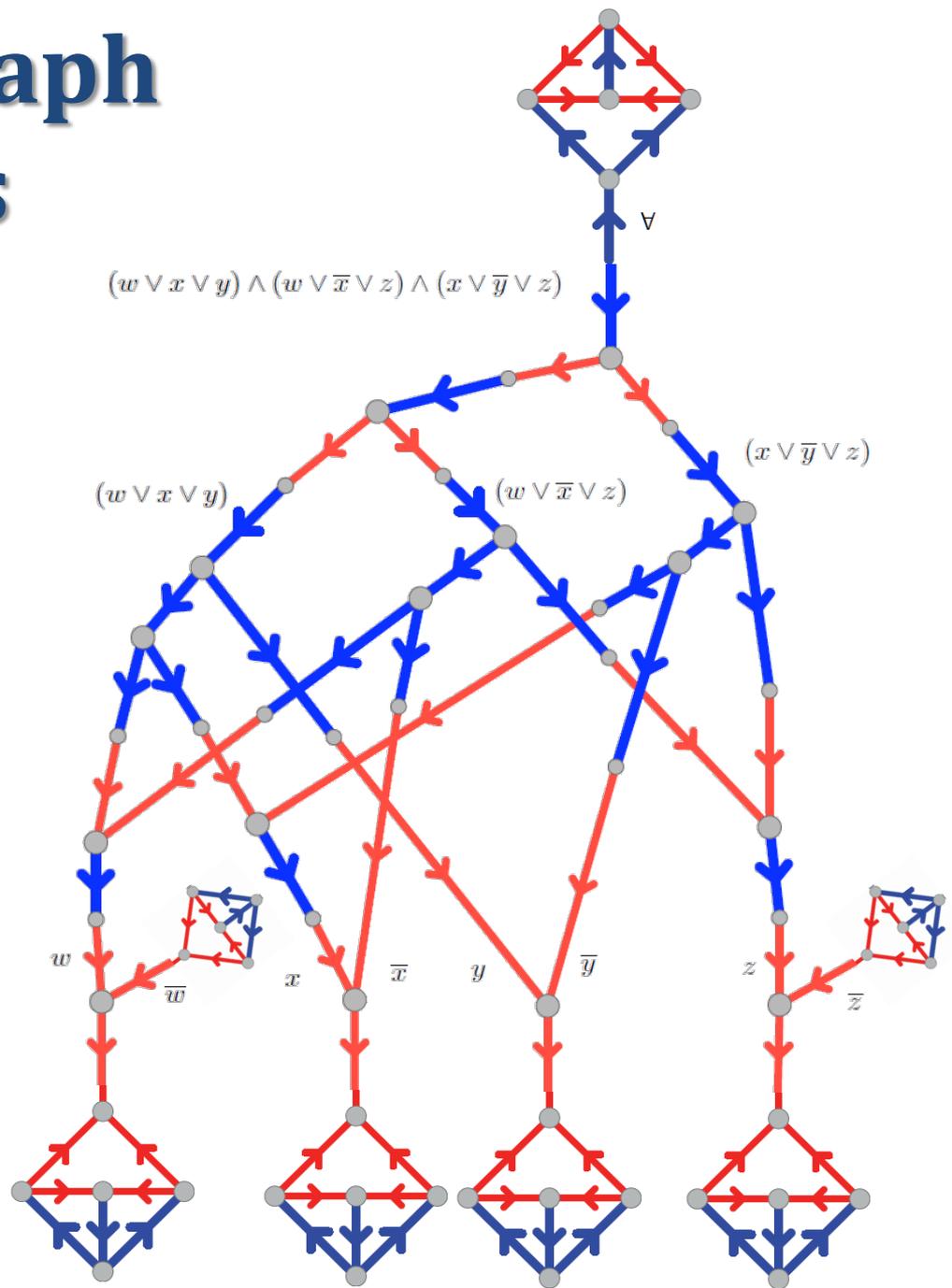
0 players
(simulation)

1 player
(puzzle)

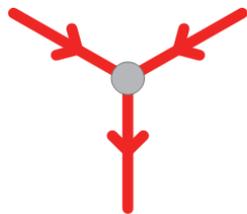
2 players
(game)

team,
imperfect info

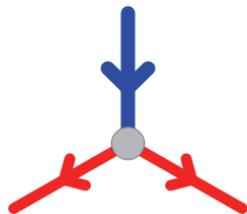
Constraint Graph Satisfaction is NP-complete



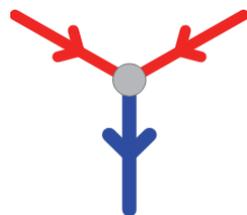
Bounded NCL is NP-complete



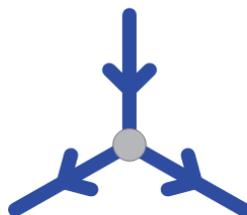
(a) CHOICE



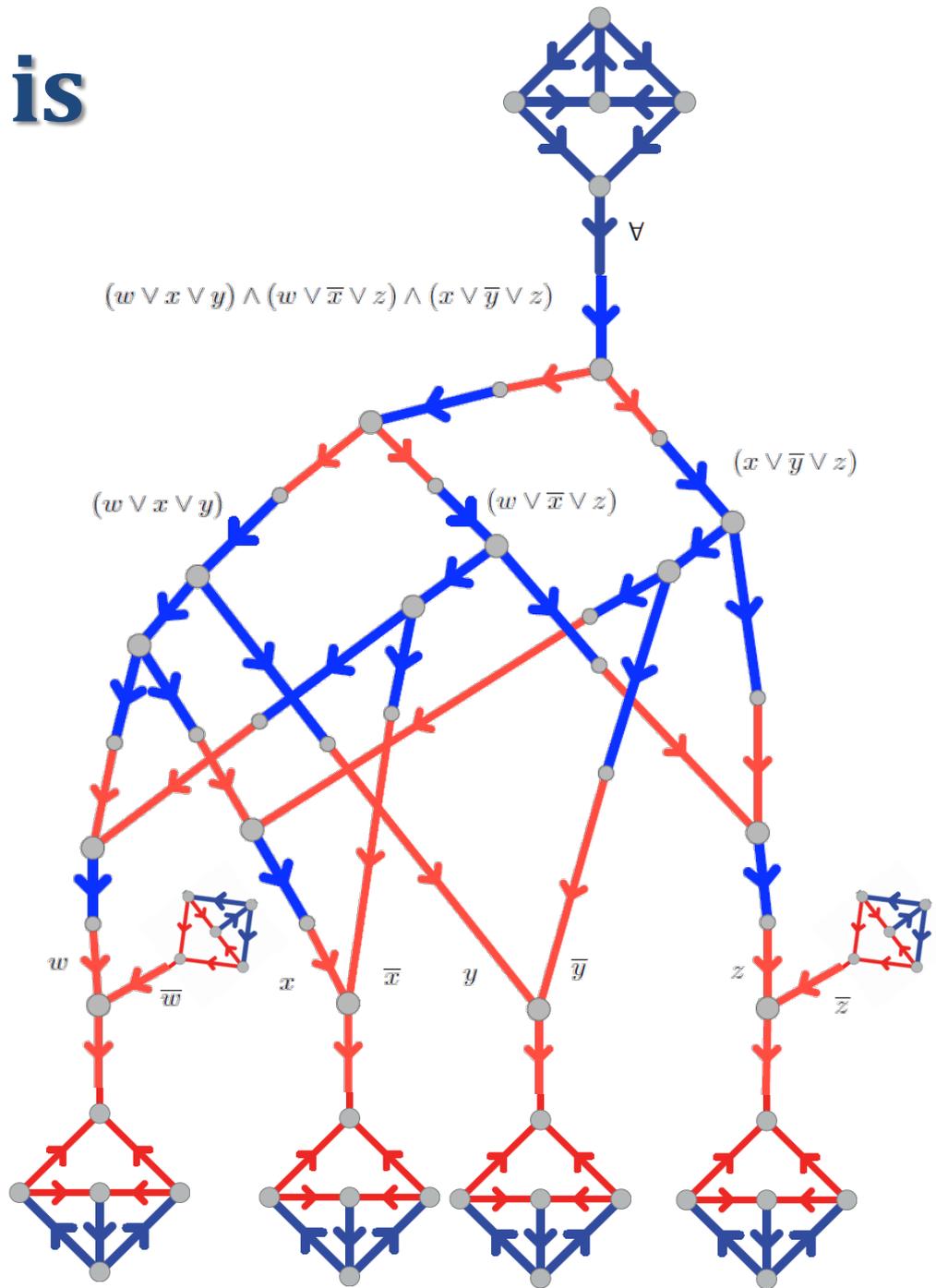
(b) AND



(c) FANOUT

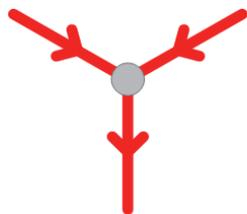


(d) OR

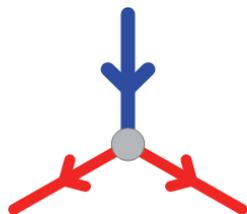




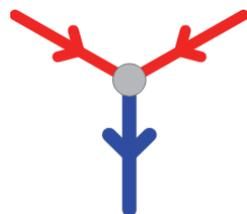
Bounded Crossover Gadget



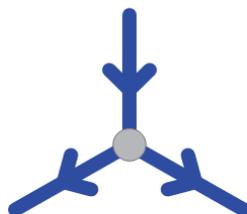
(a) CHOICE



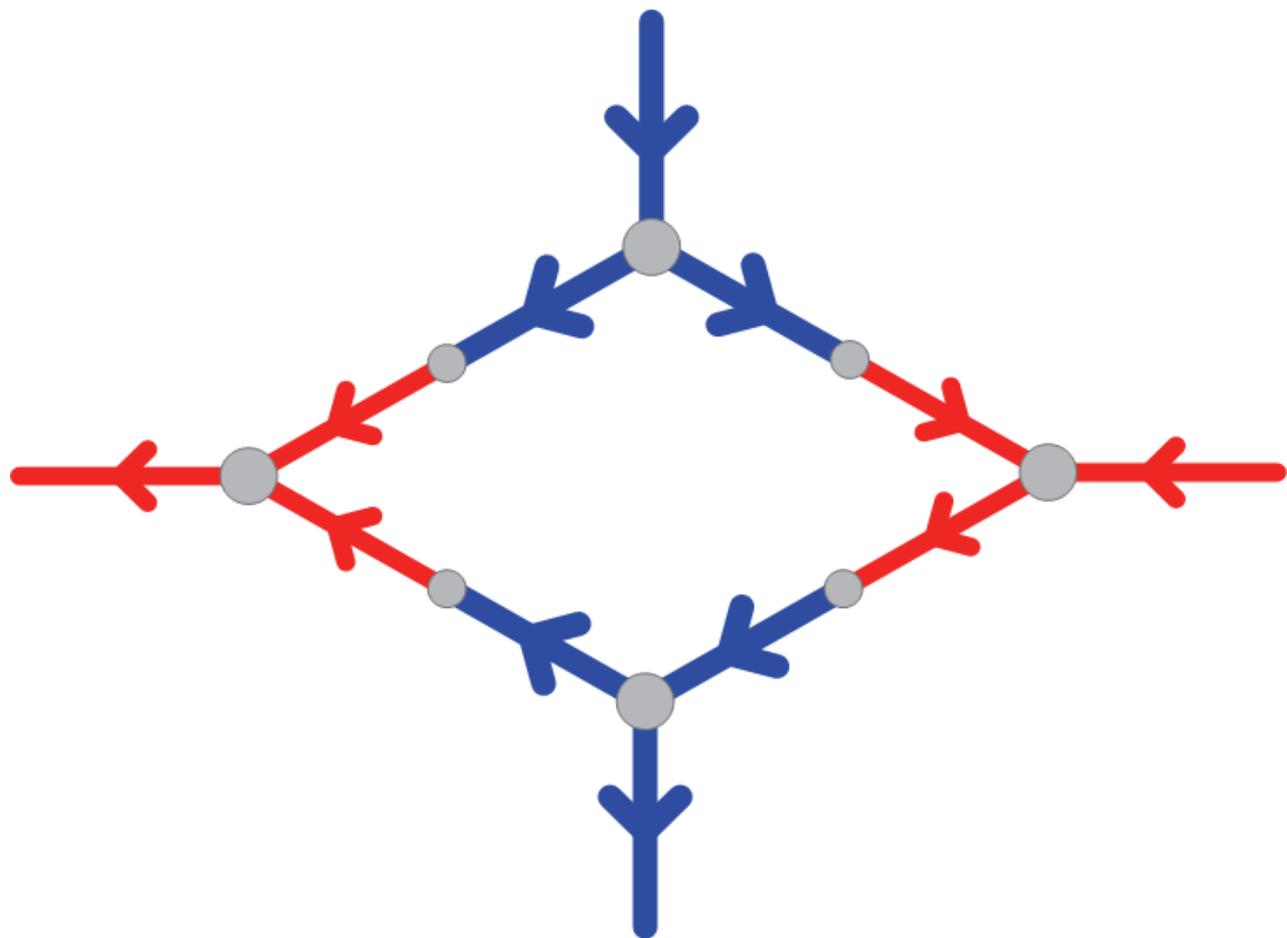
(b) AND



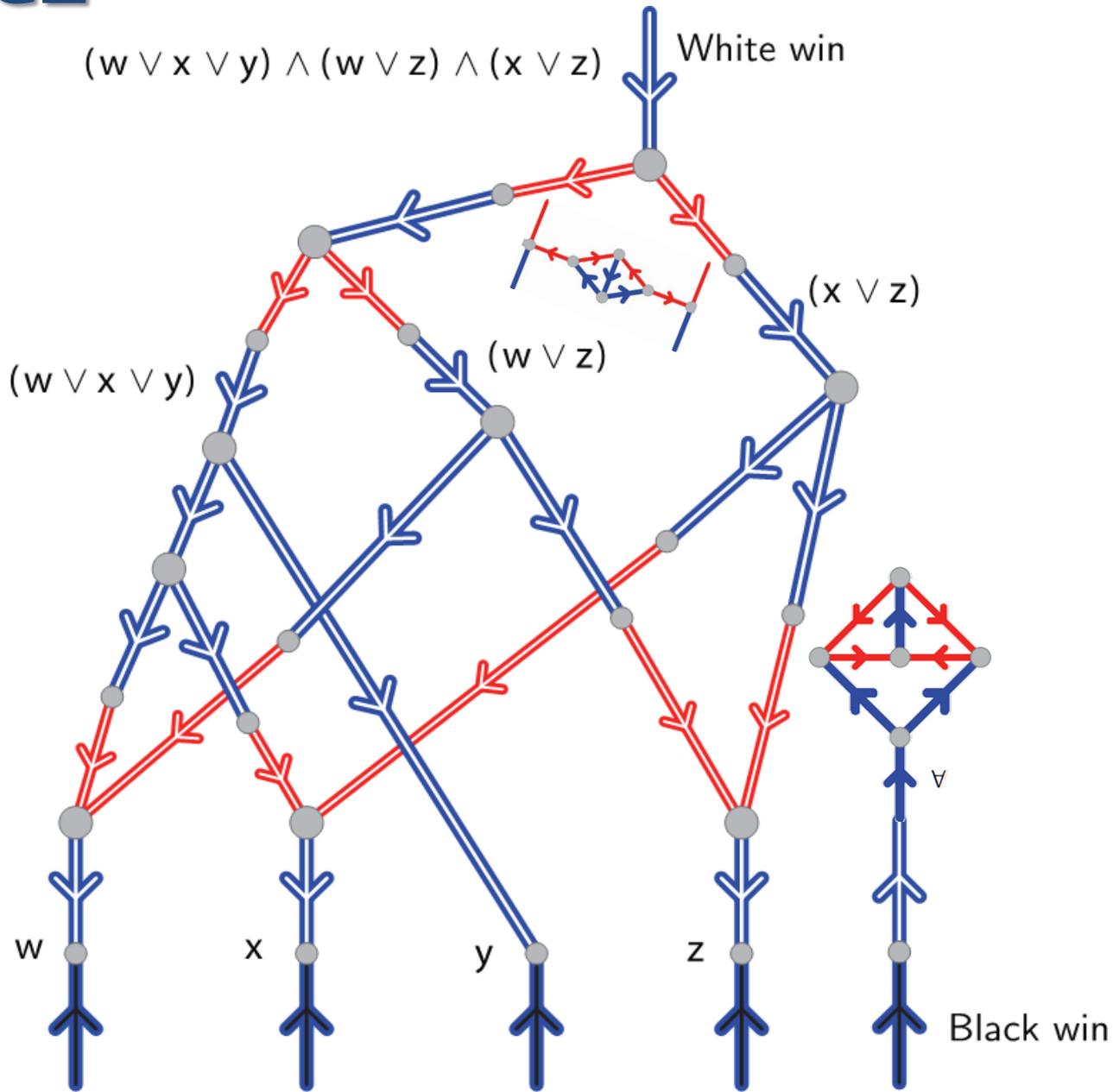
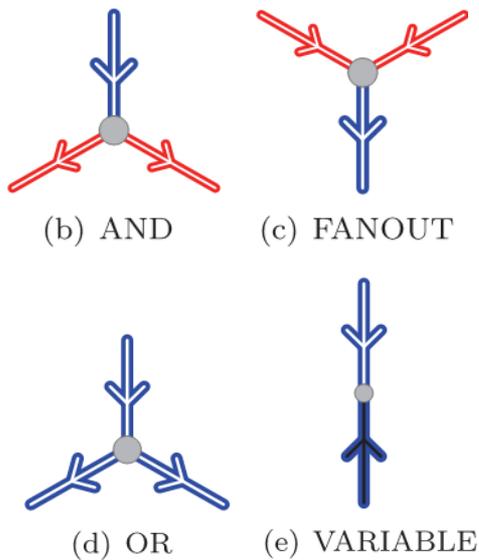
(c) FANOUT



(d) OR



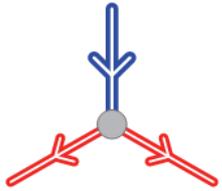
Bounded 2CL is PSPACE- complete



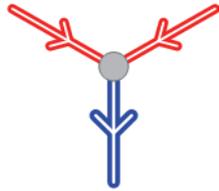
Bounded Crossover Gadget



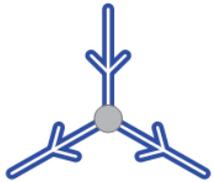
(a) CHOICE



(b) AND



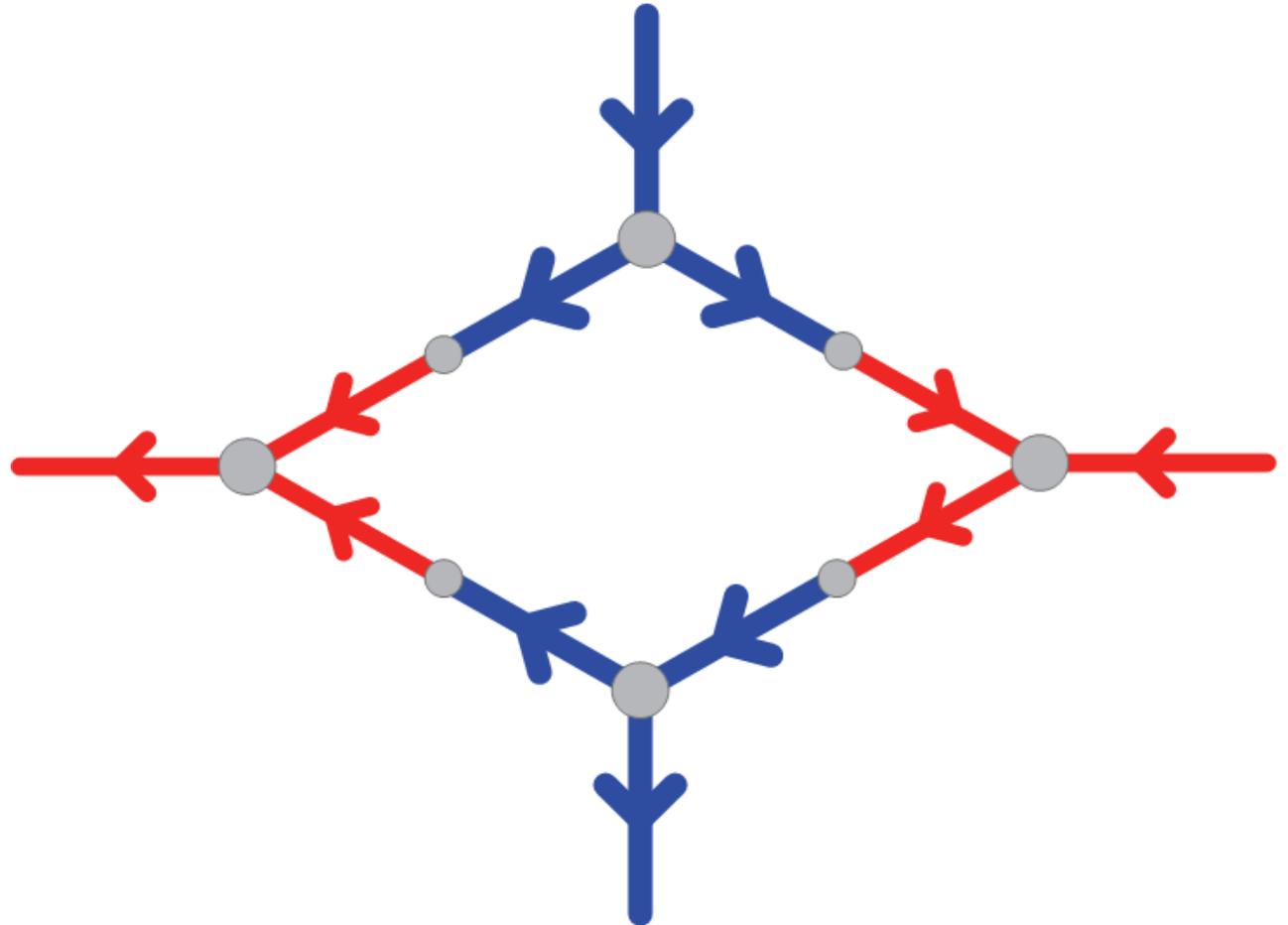
(c) FANOUT



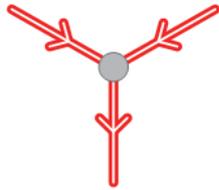
(d) OR



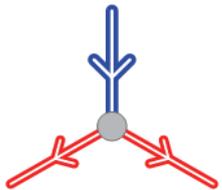
(e) VARIABLE



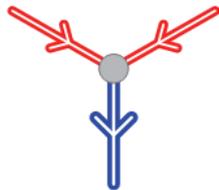
Protected OR



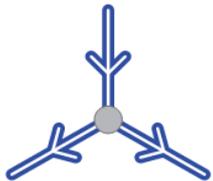
(a) CHOICE



(b) AND



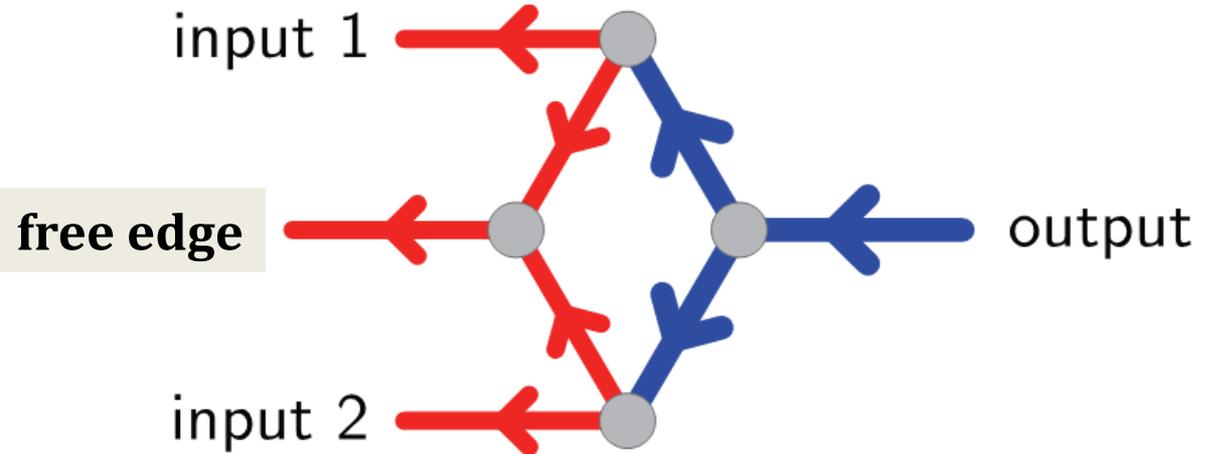
(c) FANOUT



(d) OR



(e) VARIABLE





Amazons

[Walter Zamkaskas 1988]

