6.5310: Geometric Folding Algorithms, Spring 2025
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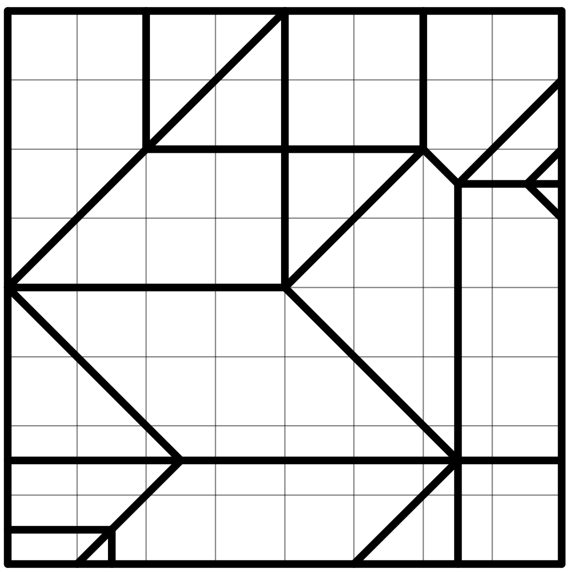
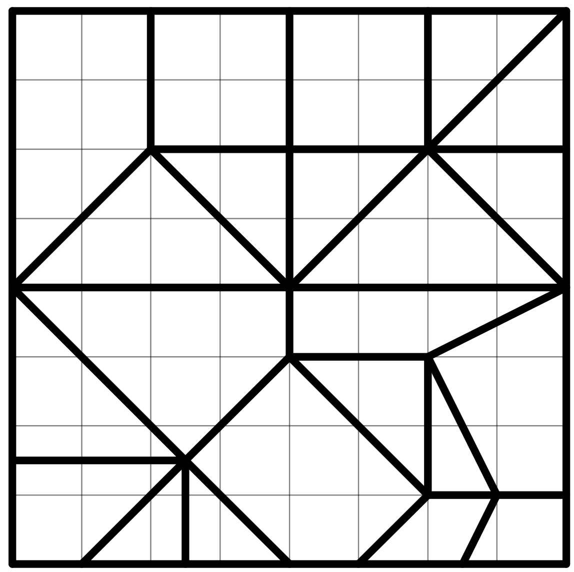
Problem Set 2 Solutions

Due: Thursday, February 20, 2025 at 10am

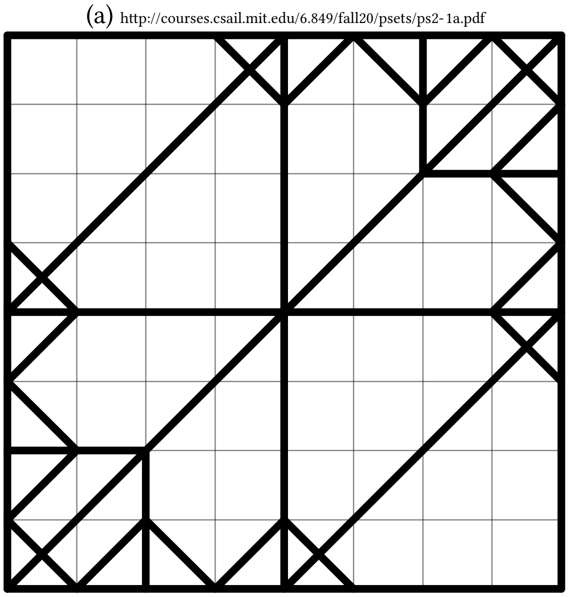
Problem 2.1 [Flat Folding]. Which of the four crease patterns in Figure 1 are flat foldable? Justify each answer by either submitting a photograph of a flat folding or arguing why the crease pattern cannot fold flat. Are any simply foldable (foldable by a sequence of simple folds)?



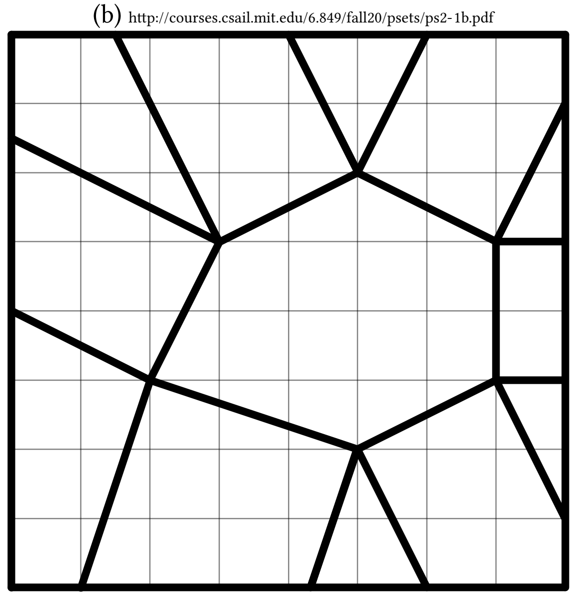
(a) http://courses.csail.mit.edu/6.849/fall20/psets/ps2-1a.pdf



(b) http://courses.csail.mit.edu/6.849/fall20/psets/ps2-1b.pdf



(c) http://courses.csail.mit.edu/6.849/fall20/psets/ps2-1c.pdf



(d) http://courses.csail.mit.edu/6.849/fall20/psets/ps2-1d.pdf

Figure 1: Crease patterns for Problem 2.1. All vertices lie on a 16×16 grid.

Optional: Is the crease pattern in Figure 2 flat foldable?

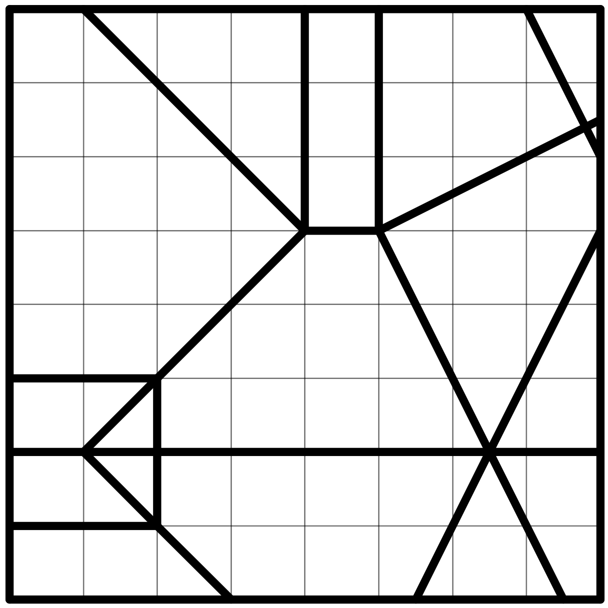


Figure 2: Optional crease pattern for Problem 2.1. http://courses.csail.mit.edu/6.849/fall20/psets/ps2-1e.pdf

Solution: Refer to Figure 3 and Figure 4 for illustrations.

(a) **Flat foldable (not with simple folds)**. This pattern is flat foldable since there is a valid MV assignment and it has a folded state without self-intersection. You can simulate the folding motion with Origami Simulator.

This pattern is not foldable with only simple folds: because there is no crease that runs straight through the center vertex, there will never be a way to locally fold that vertex using only simple folds. (Solution by Walker Anderson)

(b) **Not flat foldable**. Vertex A does not satisfy Kawasaki's theorem.

(c) **Flat foldable (with all-layer simple folds)**. Refer to Figure 3c for a sequence of simple folds.

(d) **Not flat foldable**. The MV assignments of the creases adjacent to the unique smallest angle around a degree four vertex need to be different and same for the opposite obtuse angle. There is no MV assignment that satisfies these requirements for the edges of the central hexagon.

(e) **(Optional) Not flat foldable**. Observe that the two parallel vertical creases a and b need to have different MV assignments since the paper would otherwise locally intersect. Without loss of generality, we assume $a \to M$ and $b \to V$.

Case 1: $c \to M$. Then $e, f, g \to M$ and $d \to V$. In a folded state, d and f intersect in the points highlighted in red.

Case 2: $c \to V$. Then $d, e, f \to V$ and $g \to M$. In a folded state, e and g intersect in the points highlighted in blue.

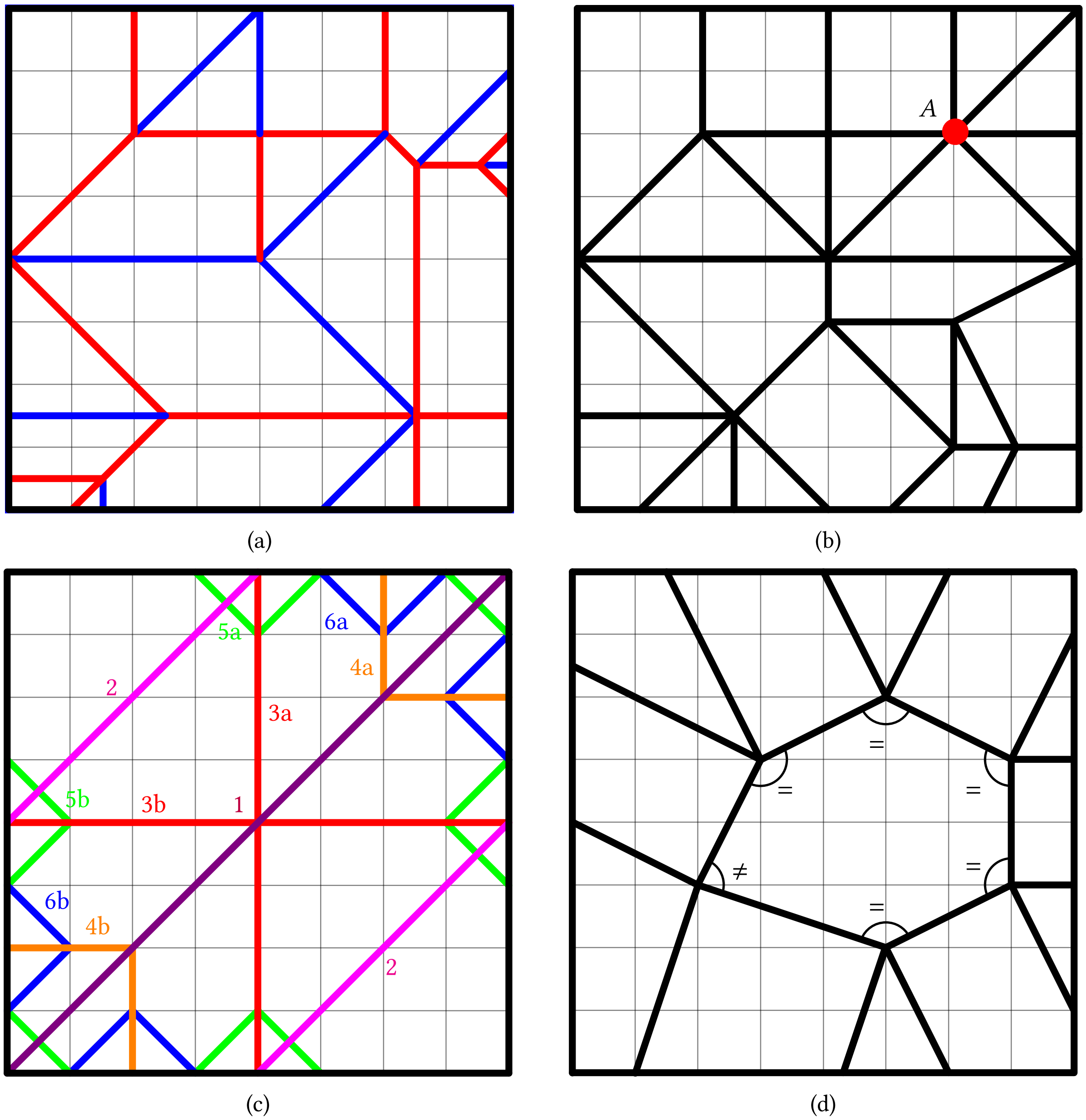


Figure 3: Solution of Problem 2.1. All vertices lie on a 16×16 grid.

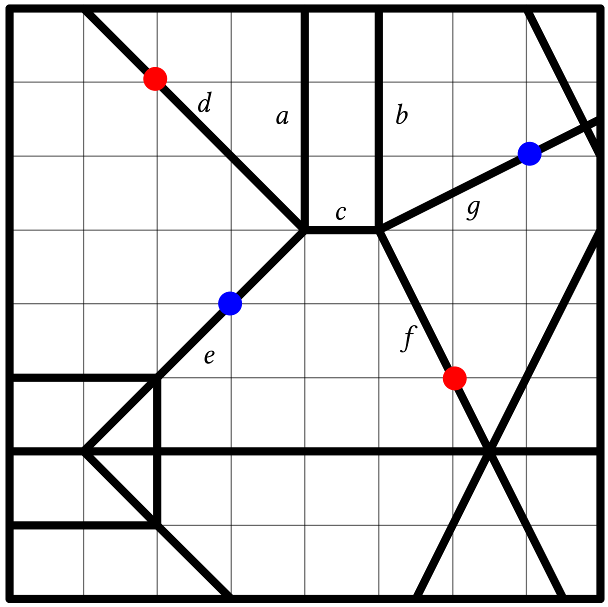


Figure 4: Solution of the optional part of Problem 2.1.

Problem 2.2 [1D Folding Counterexamples].

(a) Draw a 1D mountain-valley pattern that is mingling (as defined in Lecture 1) but not flat foldable.

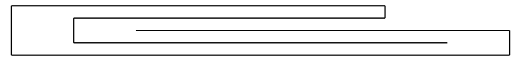
(b) Draw a folded state (including stacking order) of a 1D mountain-valley pattern that cannot be achieved by crimps and end folds.

Solution: Refer to Figure 5 for illustrations.

(a) The crease pattern in Figure 5a is mingling because the leftmost crease is closer to the end of the paper than to the second crease. If we perform the left end-fold, the pattern is no longer mingling, and therefore not flat foldable. But end-fold and crimps do not affect flat foldability, so the initial crease pattern was not flat foldable either.

(b) The folded state in Figure 5b cannot be achieved by a sequence of crimps or end-folds because there is no uncrimp or end-unfold that you can perform from this state.

(a) Mingling but not flat foldable



(b) Unreachable folded state

Figure 5: Solutions to Problem 2.