6.5310: Geometric Folding Algorithms, Spring 2025

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Problem Set 10

Due: Tuesday, April 22, 2025 at 10am

Problem 10.1 [Repeating Facet Paths]. Find a triangulated polyhedron (which has a facet path for vertex unfolding) for which *every facet path* visits at least one vertex twice (or more).

Consider a polyhedron with V vertices and F faces. Any facet path that visits all F faces exactly once must visit vertices exactly $F-1$ times. Take any polyhedron where $F-1 > V$, i.e., $F > V+1$. Then, by the Pigeonhole Principle, at least one vertex must be visited at least twice.

For example, an octahedron has 8 faces but only 6 vertices, or a triangulated cube (as pictured) has 12 faces but only 8 vertices. In both cases, $F > V+1$, so a vertex must be repeated in any facet path.

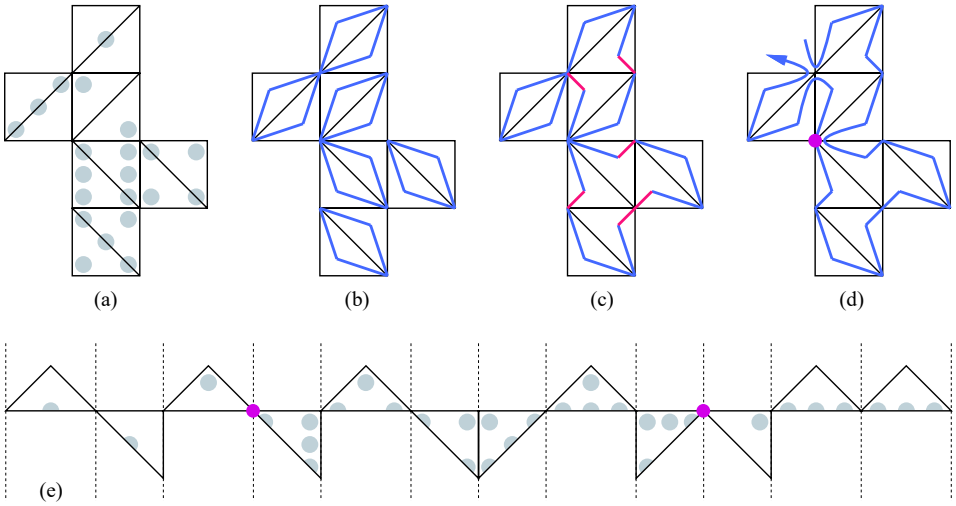

Figure 1: An example of a facet path (constructed in Lecture 17) that repeats a vertex (magenta). Based on Figure 4 of "Vertex-Unfoldings of Simplicial Manifolds" by Demaine, Eppstein, Erickson, Hart, O'Rourke (2003).