6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Recitation 4: Distinguishable strings and indices

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Problem 1: **Quiz Questions?**

Problem 2: Recall quiz question: Argue that does not exist a DFA with just 3 states that recognizes $\Sigma^*1 \cup 00*$, by showing that $\epsilon, 0, 1, 10$ must lead to different states.

In this recitation we will see a more general characterization of the minimum number of states of a machine.

Problem 3: **Equivalence Classes.** Let x and y be strings and let L be *any* language (not necessarily regular). We say that x and y are *distinguishable by* L if some string z exists such that exactly one of the strings xz and yz is in L. In the opposite case, if for all strings z, xz is in L if and only if yz is in L, we say that x and y are *indistinguishable* by L. If x and y are indistinguishable by L, we write $x \equiv_L y$.

Let L be a language and X a set of strings. We say that X is *pairwise distinguishable* by L if every two distinct strings in X are distinguishable by L. Define the *index* of L to be the maximum number of elements in any set that is pairwise distinguishable by L. In other words, the index of L is equal to the number of equivalence classes in L, which may be finite or infinite.

Let's compute indices and classes of equivalence of some languages:

1. $L_1 = (0 \cup 1)^*$.
   Answer: index is 1; the equivalence class is $(0 \cup 1)^*$
2. The language from Problem 2: $L_2 = \Sigma^*1 \cup 00*$.
   Answer: index is 4; equivalence classes: $\Sigma^*1, 00^*, \Sigma^*1\Sigma^*0, \epsilon$
3. $L_3 = (001 \cup 110)^*$.
   Answer: index is 6; equivalence classes: $(001 \cup 110)^*$, $(001 \cup 110)^*0$, $(001 \cup 110)^*00$, $(001 \cup 110)^*1$,$(001 \cup 110)^*11$, and the class formed by the rest of strings in Σ^*.

Can we build a DFA for L_3 with *less* states than the index of L?