6.045J/18.400J: Automata, Computability and Complexity	Prof. Nancy Lynch

Recitation 2: DFAs and NFAs

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Problem 1: Define the following words, phrases and symbols.

1. Finite state machine, finite automaton
2. Sipser page 49. Determinism vs Nondeterminism
3. Sipser page 54. DFA vs NFA
4. Regular Language
5. Sipser page 53. $(Q, \Sigma, \delta, q_0, F)$
6. ϕ
7. ϵ
8. Epsilon Transition
9. Sipser page 58. Regular Operations
10. Sipser page 36,40. A machines can accept many strings, but only a single language. To avoid confusion, we will usually say a machine accepts a string and recognizes a language.

Problem 2: Are the following statements true or false?

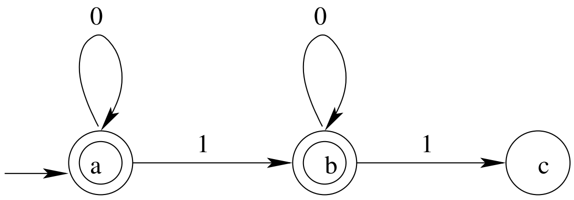
1. It is possible for a finite automaton to recognize an infinite language.
2. Every deterministic finite automaton is also a nondeterministic finite automaton.
3. For every nondeterministic finite automaton, there is an equivalent nondeterministic finite automaton that has no epsilon transitions.
4. For every nondeterministic finite automaton, there is an equivalent nondeterministic finite automaton that has a single accept state.
5. If you swap the accept states and the reject states on ANY finite automaton, the new machine will recognize the complement of the original language.
6. The class of languages recognized by non-deterministic finite automata is closed under complementation.
7. The class of languages recognized by non-deterministic finite automata is not closed under set difference.

Problem 3: Create finite automata for each of the following languages over the alphabet $\Sigma = \{0, 1\}$. Give a characterization for each state.

1. The language L_1 of strings that contain a '0' and don't end in '10'.
2. The language L_2 of strings that do not contain an odd number of 1s.
3. The languages $L_1 \cup L_2$, $L_1 \cap L_2$, $L_2 | L_1$, L_2^*.

Problem 4: Show that the language $L_5 = \{s \in \{0,1\}^* \mid s \text{ divisible by } 5\}$ is regular.

Problem 5: Let's prove that the following automaton recognizes exactly the language $L = \{w \in \{0,1\}^* | w \text{ contains less than 2 ones}\}$. To do this, we will need to prove that our FA (1) accepts all strings in L and (2) does not accept any string not in L.



1. Characterize each state.

2. Forward direction (accepts all strings in L). Proof by Induction?

3. Reverse direction (does not accept any string outside of L). Proof by Contradiction?

Problem 6:(Closure of regular languages under perfect shuffle) (Sipser 1.41)
For languages A and B, let the **perfect shuffle** of A and B be the language:

$$\{w \mid w = a_1 b_1 a_2 b_2 \ldots a_k b_k, \text{ where } a_1 a_2 \ldots a_k \in A, b_1 b_2 \ldots b_k \in B.\}$$

Show that regular languages are closed under the perfect shuffle operation.