6.045J/18.400J: Automata, Computability and Complexity Nancy Lynch

Homework 9

Due: Wednesday, April 18, 2007, 5PM *Elena Grigorescu*

Readings: Sections 7.1, 7.2, 7.3

Problem 1: (Adapted from Sipser Problems 7.1 and 7.2) Answer each of the following with TRUE or FALSE. You do not need to justify your answers. (Note: when dealing with sets like $O(f(n))$, $\Omega(f(n))$, etc., we use the symbols $=$ and $\in$ interchangeably.)

1. $5 = O(n)$
2. $7n = O(n)$
3. $n^3 = O(n^2 \log^2(n))$
4. $n \log(n) + 10n = O(n^2)$
5. $4^n = O(2^n)$
6. $3^n = 2^{O(n)}$
7. $2^{2^n} = O(2^{2^n})$
8. $n^n = O(n!)$
9. $n = o(2n)$
10. $2n = o(n^2)$
11. $2^n = o(3^n)$
12. $1 = o(n)$
13. $n = o(\log(n))$
14. $\frac{1}{3} = o(1)$
15. $log_2(n) = \Theta(log_3(n))$
16. $2^n = \Theta(3^n)$
17. $n^5 = \Theta(32^{log_2(n)})$
18. $n^3 = \Omega(n^4)$
19. $log(n) = \Omega(log(log(n)))$
20. $3^{2^n} = \Omega(2^{3^n})$

Problem 2: Prove that P is closed under the following operations:

(a) union,
(b) intersection,
(c) complement,
(d) concatenation.

P is also closed under the star operation, but that is a bit harder to show. (see problem 7.14).

Problem 3: Prove that NP is closed under the following operations:

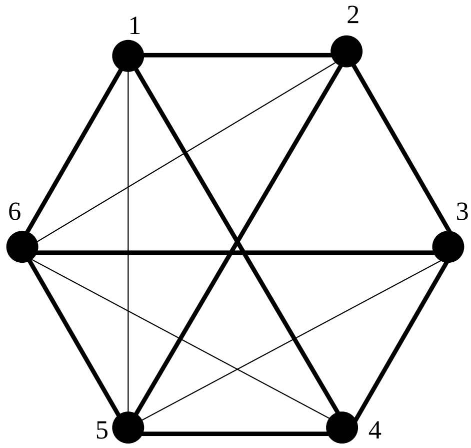
(a) union,
(b) intersection,
(c) concatenation.

NP is also closed under star (see the solution to problem 7.15), but it is not known whether NP is closed under complement.

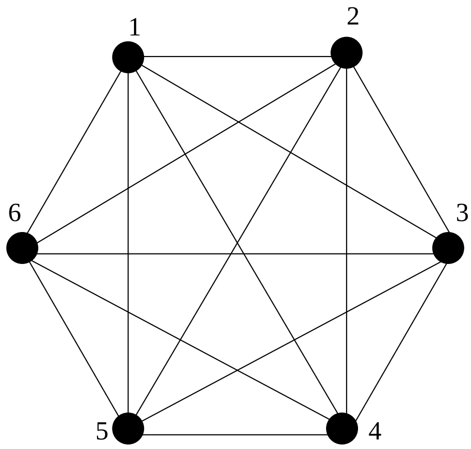
Problem 4: Prove that the following languages are in NP. You may use either the guess-and-check (certificate/verifier) method, or else describe a nondeterministic Turing machine that decides the language in time polynomial in the length of the input.

1. NO-TRIANGLES = $\{\langle G\rangle \mid G = (V, E)$ is an undirected graph whose edge set E can be partitioned into two disjoint sets E_1 and E_2 so that neither graph (V, E_1) nor (V, E_2) contains a triangle$\}$.

For example, the following graph is in NO-TRIANGLES (the edges can be split into two graphs such that neither contains a triangle; let the bold edges be in E_1 and the others in E_2):



The following graph is not in NO-TRIANGLES:



2. BOUNDED-PCP (Bounded Post Correspondence Problem), for a fixed alphabet Σ with $|\Sigma| \geq 2$. This is defined as $\{S, k|\ S$ is a finite set of dominoes over Σ, k is an integer written in unary, and there is a sequence of at most k dominoes (allowing repeats) for which the top and bottom sequences are equal$\}$.

   If k was not written in unary, would your solution to the above still work? Why or why not?