Reading: Handout 6 (from Hopcroft book); Sipser, Section 6.3

For the final two problems of this unit, you will have to consider some new definitions that were not covered in class, but are presented in Section 6.3 of Sipser's book. Namely, an *Oracle Turing Machine* is a variant of a Turing machine that has the ability to query an external source – an "oracle" – about membership of a string in some particular *oracle language*. A language is *decidable relative to* a language L if it can be decided by an oracle Turing machine that uses L as its oracle set.

Problem 1: (From Sipser Problem 6.19.)
Recall the Post Correspondence Problem discussed in class and in Section 5.2 of Sipser. Show that PCP is decidable relative to A_{TM}, the acceptance problem for ordinary Turing machines.

Problem 2: (From Sipser Exercise 6.4.)
Let $A'_{TM} = \{\langle M, w \rangle \mid M$ is an oracle Turing machine and $M^{A_{TM}}$ accepts $w\}$. Thus, A'_{TM} can be thought of as the "acceptance problem for oracle TMs relative to the acceptance problem for ordinary TMs".

Show that A'_{TM} is not decidable relative to A_{TM}. That is, even with an oracle for the ordinary acceptance problem, the relative version of the acceptance problem still cannot be decided!

(Hint: Use a diagonalization argument like the one used to prove undecidability of A_{TM}.)