**6.045J/18.400J: Automata, Computability and Complexity** Nancy Lynch

# Homework 7

*Due: Monday, April 2* *Elena Grigorescu*

**Readings:** Handout 6 (from Hopcroft book); Section 5.3; Problems 5.28-5.30.

**Problem 1**:
(Counter Machine Examples)
Informally but clearly describe counter machines that recognize the following languages. Use as few counters as you can. (You may assume that a special endmarker appears at the end of the input string.)

1. $\{a^i b^j | i \leq j\}$.
2. $\{a^i b^j c^k | i \leq j \leq k\}$.
3. $\{a^i b^j c^k | k = ij\}$.

**Problem 2**: (Counter Machine Simulation of Stack Machine)
Suppose we design a counter machine to simulate a $k$-stack machine, using the same strategy discussed in lecture for simulating 2 stacks with 3 counters.

1. How many counters does this require? Why?
2. Simulating a single step of the $k$-stack machine requires that the CM (Counter Machine) emulate the reading of the top symbol of each stack, without removing that symbol from the stack. Describe clearly how the CM can read the top symbol of a stack.
3. Before the CM can determine the stack machine transition to be emulated, it must know the top symbol of every stack. Describe how the CM can remember these symbols in a way that allows it to determine the next stack machine transition.
4. After the CM determines the stack machine transition to be simulated, it must modify the simulated stacks as specified by that transition. Describe clearly how the CM can do this. How can it keep track of its progress?
5. How does the CM simulate the state change of the simulated stack machine?

**Problem 3**: (Mapping Reducibility)
Define *Total* to be the language $\{< M > \mid M \text{ is a Turing machine and } L(M) = \{0,1\}^*\}$, that is, the set of representations of (basic) Turing machines that accept all strings of 0s and 1s. Use mapping reducibility to prove the following statements. Cite explicitly any theorems about mapping reducibility that you use.

1. The complement of *Total* is not Turing-recognizable.
2. *Total* is not Turing-recognizable.

**Problem 4**: (Rice's Theorem Applications)
Which of the following are decidable and which are undecidable? Give proofs. Use Rice's Theorem wherever you can, in showing undecidability.

1. $\{< M > \mid M$ is a Turing machine and $M$ has more than 75 states$\}$.
2. $\{< M > \mid M$ is a Turing machine and $L(M)$ is recognized by some Turing machine $M'$ with more than 75 states$\}$.
3. $\{< M > \mid M$ is a Turing machine and $L(M)$ is recognized by some Turing machine $M'$ with at most 75 states and whose tape alphabet contains only five symbols$\}$.
4. $\{< M > \mid M$ is a Turing machine that accepts every even-length string (and possibly other strings)$\}$.
5. $\{< M_1, M_2 > \mid M_1$ and $M_2$ are Turing machines and $L(M_1) \cap L(M_2) = \emptyset\}$.