6.045J/18.400J: Automata, Computability and Complexity Nancy Lynch

Homework 6

Due: March 19, 2007 *Elena Grigorescu*

Problem 1: Prove that the following languages are undecidable. Use reductions from A_{TM} or other problems already known to be undecidable. Here, $\Sigma = \{0, 1\}$.

1. $L_1 = \{< M >: M$ is a Turing machine and M accepts the empty string $\epsilon\}$.
2. $L_2 = \{< M >: M$ is a Turing machine and $L(M)$ is infinite $\}$.

Problem 2: (From Sipser problems 4.19 and 5.9)

1. Let $S = \{< M > | M$ is a DFA that accepts w^R whenever it accepts $w\}$. Prove that S is decidable.
2. Let $T = \{< M > | M$ is a basic Turing machine that accepts w^R whenever it accepts $w\}$. Prove that T is undecidable.

Problem 3: (From Sipser, problem 4.28)
Let $A = \{< D_1 >, < D_2 >, < D_3 >, \ldots\}$ be an infinite language consisting of representations of Turing machines that are deciders, that is, each machine D_i halts (accepts or rejects) on every input. Suppose that A is Turing-recognizable, and therefore, enumerable by an enumerator machine E.

Show that there must be some decidable language that is not decided by any of the machines represented in A (i.e., some language $L(D')$ that is decided by a machine D' such that $< D' > \notin A$).

Note that this time, countability arguments are not going to help: there are only countably many machine descriptions in A, but then again, there are also only countably many decidable languages. Still, A cannot contain descriptions of deciders for *all* decidable languages.

(Hint: Recall the diagonalization method; try constructing D' using the enumerator E.)

Problem 4: Consider the machine M_2 on page 143 of Sipser's book, which recognizes the language consisting of all strings of 0s whose length is a power of 2.

1. Write out the accepting computation history for the machine M_2 on input 00.
2. What are the dominoes for the instance of the Post Correspondence Problem defined for M_2 and input 00?
3. Write out your computation history from part (a) twice, one copy above the other. Draw lines indicating how your dominoes from part (b) can be used to establish a correspondence between these two copies.