6.045J/18.400J: Automata, Computability and Complexity	Prof. Nancy Lynch
Homework 3	
Due: Monday, February 26, 2007, 5PM	*Elena Grigorescu*

Reading: Sipser, Sections 1.3 and 1.4

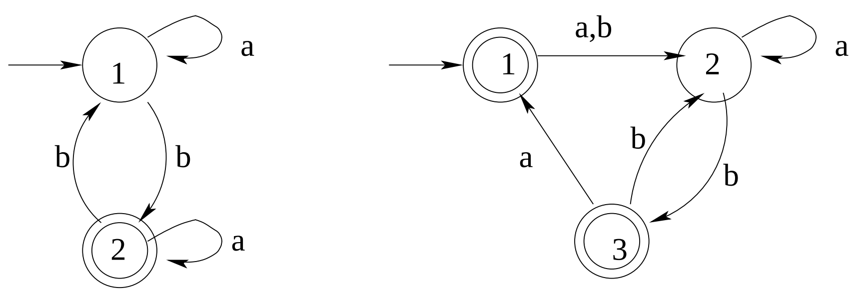
Problem 1: Taken from Sipser 1.18. Give regular expressions generating the following languages. In all cases the alphabet is $\{0, 1\}$.

1. $L_1 = \{w|w \text{ contains at least three } 1s\}$.
2. $L_2 = \{w|w \text{ has length at least 3 and its third symbol is } 0\}$.
3. $L_3 = \{w|w \text{ doesn't contain the substring } 110\}$.
4. $L_4 = \{w| \text{ every odd position of } w \text{ is a } 1\}$.
5. $L_5 = \{w|w \text{ contains at least two } 0s \text{ and at most one } 1\}$.

Problem 2: Sipser 1.19. Use the procedure described in Lemma 1.55 to convert the following regular expressions to nondeterministic finite automata.

1. $(0 \cup 1)^*000(0 \cup 1)^*$
2. $(((00)^*(11)) \cup 01)^*$
3. $\emptyset^*$

Problem 3: Sipser 1.21. Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.



(a) (b)

Problem 4: Use the pumping lemma to show that the following languages are not regular.

1. $A_1 = \{www|w \in \{0, 1\}^*\}$.
2. $A_2 = \{w \in \{0, 1\}^*| \text{ the number of } 0s \text{ in } w \text{ is a perfect square } \}$.

Problem 5: Based on Sipser 1.30. Describe the error in the following "proof" that 0^*1^* is not a regular language. (An error must exist because 0^*1^* *is* regular.)
The proof is by contradiction. Assume that 0^*1^* is regular. Let p be the number of states in a DFA recognizing 0^*1^*. Choose s to be the string 0^p1^p. You know that s is a member of 0^*1^*, but Example 1.73 (in the text) shows that s cannot be pumped. Thus you have a contradiction. So 0^*1^* is not regular.

Problem 6: Sipser 1.47
Let $\Sigma = \{1, \#\}$ and let

$$Y = \{w \mid w = x_1\#x_2\#\ldots\#x_k \text{ for } k \geq 0, \text{ each } x_i \in 1^*, \text{ and } x_i \neq x_j \text{ for } i \neq j\}.$$

Prove that Y is not regular.