6.045J/18.400J: Automata, Computability and Complexity	Prof. Nancy Lynch

Homework 2

Due: Tuesday, February 20, 2007 — *Elena Grigorescu*

Reading: Sipser, Sections 1.2 and 1.3.

Problem 1: **Designing DFAs and NFAs.** For each of the following, draw a state diagram for a DFA or NFA (as required) that recognizes the specified language. In all cases the alphabet is $\{0, 1\}$.

(a) Sipser exercise 1.6, part c. $L_1 = \{w | w \text{ contains the substring } 0101\}$. Provide a DFA recognizing L_1.

(b) Sipser exercise 1.6, part j. $L_2 = \{w | w \text{ contains at least two } 0s \text{ and at most one } 1\}$. Provide a DFA recognizing L_2.

(c) Sipser exercise 1.7, part c. $L_3 = \{w | w \text{ contains an even number of } 0s, \text{ or exactly two } 1s\}$. Provide an NFA with at most six states that recognizes L_3.

Problem 2: **Proving an FA recognizes a language.** For one of the automata you designed in problem 1, prove that the machine recognizes *exactly* the specified language. To do this, you will need to prove that your automaton (1) accepts all strings in the language and (2) does not accept any string not in the language.

Problem 3: **A harder DFA design problem.** Sipser problem 1.33.
Let

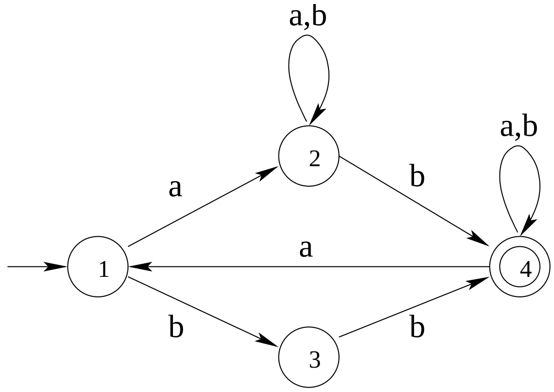
$$\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

Here, Σ_2 contains all columns of 0s and 1s of height two. A string of symbols in Σ_2 gives two rows of 0s and 1s. Consider each row to be a binary number and let

$$C = \{w \in \Sigma_2^* | \text{ the bottom row of } w \text{ is three times the top row}\}.$$

For example, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} \in C$, but $\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \notin C$. Provide an NFA that recognizes C.

Problem 4: **NFA to DFA.**
Consider the following state diagram.



(a) The state diagram above represents an NFA $N = (Q, \Sigma, \delta, q_0, F)$. Say what each of the components of the 5-tuple is, for this NFA.

(b) Apply the subset construction described in class to obtain a DFA $M = (Q', \Sigma, \delta', q_0', F')$ that is equivalent to N. Define the components of M, Q', δ', q_0', F' precisely. You may describe δ' via either a transition table or a state diagram.

Problem 5: Regular languages are closed under the star operation. Sipser exercise 1.15. Give a counterexample to show that the following construction fails to prove Theorem 1.49, the closure of the class of regular languages under the star operation.[1] Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be an NFA that recognizes A_1. Construct NFA $N = (Q_1, \Sigma, \delta, q_1, F)$ as follows. N is supposed to recognize A_1^*.

(a) The states of N are the states of N_1.

(b) The start state of N is the same as the start state of N_1.

(c) $F = \{q_1\} \cup F_1$. The accept states F are the old accept states plus its start state.

(d) Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \notin F_1 \text{ or } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon. \end{cases}$$

(Suggestion: convert this formal construction to a picture, as in Figure 1.50 of Sipser.)

Problem 6: Showing closure under other operations. In each of the following parts we define an operation on a language A. Show that the class of regular languages is closed under that operation.

(a) $\mathit{Reverse}(A) = \{w \mid \text{the reverse of } w \text{ is in } A\}$.

(b) $\mathit{Extend}(A) = \{w \mid \text{some prefix of } w \text{ is in } A\}$. (A string x is a **prefix** of string y if a string z exists where $xz = y$.)

(c) $\mathit{Suffix}(A) = \{w \mid w \text{ is a suffix of some string in } A\}$. (A string x is a **suffix** of string y if a string z exists where $zx = y$.)

[1] In other words, you must present a finite automaton, N_1, for which the constructed automaton N does not recognize the star of N_1's language.