6.045J/18.400J: Automata, Computability and Complexity	Nancy Lynch
Homework 10	
Due: April 23, 2007	*Elena Grigorescu*

Readings: Sections 7.4, 7.5

Problem 1: Let A and B be nontrivial languages over an alphabet Σ (that is, not equal to $\emptyset$ or Σ^*). State whether each of the following is KNOWN TO BE TRUE, KNOWN TO BE FALSE, or UNKNOWN. Explain carefully why. For example if you claim that a reduction exists, then you should actually define the reduction.

1. If $A \leq_P B$, then $\overline{A} \leq_P \overline{B}$.
2. If $B \in P$ and A is nontrivial (not equal to $\emptyset$ or $Sigma^*$), then $A \cap B \leq_P A$.
3. If $B \in P$ and A is nontrivial, then $A \cup B \leq_P A$.
4. If $A \cap B$ is NP-complete, $A \in NP$ and $B \in P$, then A must be NP-complete.
5. If $A \cup B$ is NP-complete, $A \in NP$ and $B \in P$, then A must be NP-complete.
6. If A is NP-complete, $\overline{A} \in NP$ and $B \in NP$, then $\overline{B}$ must be in NP.
7. If A is NP-complete, $\overline{A} \in NP$ and $B \in NP$, then $\overline{B}$ must be in P.

Problem 2: For each of the following pairs of sets A and B, show that $A \leq_P B$.

1. $A = SAT$, and $B = SAT\text{-}UNSAT = \{\langle\phi\rangle \mid \phi$ is a satisfiable Boolean formula that is not a tautology (that is, it has at least one non-satisfying assignment)$\}$;
2. $A = SAT$, and
   $B = TRIPLE - SAT = \{\langle\phi\rangle \mid \phi$ is a satisfiable Boolean formula that has at least three distinct satisfying assignments $\}$.
3. $A = VC$, the Vertex Cover problem, and
   $B = HALF{-}VC$, defined as $\{\langle G\rangle \mid G$ is an undirected graph with an even number of vertices, of which some half form a vertex cover $\}$.