6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Homework 7.1 (FAKE)

Due: NEVER *Elena Grigorescu*

This "fake" homework is intended simply as a study guide for the material covered in class 14, on Monday, April 2.

Readings: Section 6.1

Problem 1: (Recursion Theorem application)

1. Write a recursive program that computes the reversal of its input string x. This should be similar to the program P_2 given in class, which computes the length of the input string.
2. Use the Recursion Theorem to justify why your program can be emulated by an ordinary Turing machine. Say what T, R, t, and r correspond to in this application of the Recursion Theorem, and show that any hypotheses needed to apply the theorem are satisfied.

Problem 2: (Using the Recursion Theorem to prove undecidability)

Let

$$A01 = \{\langle M\rangle |\ M \text{ is a Turing machine and } M \text{ accepts the string } 01\}$$

be the language of descriptions of machines accepting the string 01. We already know how to prove that $A01$ is undecidable using mapping reducibility or using Rice's Theorem. Now give yet another proof that $A01$ is undecidable, by using the Recursion Theorem.

Problem 3: (Using the Recursion Theorem to prove non-enumerability)

Let's define a Turing machine M to be *pretty good* if the length of the representation of every Turing machine M' that recognizes the same language as M is at least the square root of the length of the representation of M. (That is, for every M' such that $L(M') = L(M)$, $| < M' > | \geq \sqrt{| < M > |}$. Prove that there is no enumerator that outputs the set of representations of pretty-good Turing machines.