6.045J/18.400J: Automata, Computability and Complexity T.A. Matthew Lepinski

Recitation 7: Reducibility

March 20, 2003 *Discussion Problems*

Problem 1: Mapping Reducibility, $\leq_M$, is a relation on languages:

- Is the $\leq_M$ relation Reflexive?
- Is the $\leq_M$ relation Symmetric?
- Is the $\leq_M$ relation Transitive?

Problem 2: Use Rice's Theorem to prove that some language, L, is undecidable.

Problem 3: Use Mapping Reducibility to show that the following language is undecidable:

$$L = \{< M, N > | M \text{ and } N \text{ are Turing Machines such that } L(M) \subseteq L(N)\}$$

Problem 4: Prove Rice's Second Theorem:
For any non-monotone property, P, of recognizable languages, the following language is NOT recognizable:

$$A_P = \{< M > | L(M) \text{ has property, P}\}$$

Note: A property of recognizable languages is, P, is non-monotone if there exists a pair of recognizable languages A and B such that $A \subseteq B$, $P(B) = TRUE$ and $P(A) = FALSE$.

Problem 5: Show that the halting problem for single counter machines is decidable.
That is show the following language is decidable:

$$HALT_{1C} = \{< C, x > | C \text{ is a counter machine that halts on input x}\}$$