6.045J/18.400J: Automata, Computability and Complexity	T.A. Matthew Lepinski

Recitation 1: Math Review

February 6, 2003 — *Discussion Problems*

Problem 1: Define the following words, phrases and symbols.

1. Set
2. Cardinality
3. Relation
4. Reflexive (Relation)
5. Symmetric (Relation)
6. Transitive (Relation)
7. Equivalence Relation
8. Alphabet (Input Alphabet/Output Alphabet)
9. String
10. Machine Accepts a String
11. Language
12. Language Recognized by a Machine (Language Accepted by a Machine)
13. Finite State Machine (Finite State Automata)
14. $p \Rightarrow q$
15. Logically Equivalent
16. Proof by Contradiction
17. Proof by Induction (Base Case) (Induction Hypothesis) (Inductive Step)

Problem 2: Prove by Contradiction: If there are 6 people at a party shaking hands, then there most be at least two people who shook hands with the same number of other people.

Problem 3: Problem 0.11 from Sipser's Text.
Find the Error in the following proof that all horses are the same color.
Claim: In any set of h horses, all horses in the set are the same color.
Proof: By induction on h.
Basis: For $h = 1$. In any set containing just one horse, all horses clearly are the same color.
Inductive Step: For $k \geq 1$, assume that the claim is true for $h = k$ and prove that it is true for $h = k + 1$. Take any set H of $k + 1$ horses. we will show that all horses in this set are the same color. Remove one horse from this set to obtain the set H_1 with just k horses. By the induction hypothesis, all the horses in H_1 are the same color. Now replace the removed horse and remove a different one to obtain the set H_2. By the same argument, all the horses in H_2 are the same color. Therefore, all the horses in H must be the same color and the proof is complete.