Handout 15: Quiz 3

Write your full name on each page.

Name:

Problem 1: **Trick Questions: (3 points each)**

1. True or false: If all the functions $f_1, f_2, f_3 \cdots$ are all polynomial time computable, then the function $g(x) \stackrel{\text{def}}{=} f_1(f_2(\cdots f_{|x|}(x)\cdots))$ (i.e., on input of length n we apply the first n functions) is also polynomial time computable. Briefly explain your answer.

2. Give a language $L \in \mathcal{NP}$ such that $L \leq_p \overline{L}$.

3. True or false: If $A \leq_p B$, $B \leq_p C$ and both A and C are $\mathcal{NP}$-complete, than B is $\mathcal{NP}$-complete. Briefly explain your answer.

4. True or false: The problem of determining if a graph G has a 7-clique is $\mathcal{NP}$-complete. Briefly explain your answer.

5. True or false: Given a set S of integers, the problem of deciding if it can be partitioned into 3 disjoint subsets that share the same sum is $\mathcal{NP}$-complete. Briefly explain your answer.

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Problem 2: **(15 points)** Show the following language to be $\mathcal{NP}$-complete:

$$GRAPH - EMBED = \left\{ \langle G_1, G_2 \rangle \ : \ \begin{array}{l} G_1 \text{ and } G_2 \text{ are undirected graphs and } G_2 \text{ can be} \\ \text{embedded in } G_1 \end{array} \right\}$$

(A graph B can be embedded in a graph A iff B can be made into a sub-graph of A simply by relabeling the nodes. That is, B can be embedded in A iff there exists a function f from nodes in B to nodes in A such that there is an edge between nodes n_1 and n_2 in B iff there exists an edge between $f(n_1)$ and $f(n_2)$ in A.) Hint: Consider $CLIQUE$.

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Problem 3: **(15 points)** The *SHOPLIFTER* problem is as follows: you have before you a set of objects n_1, n_2, $\ldots n_k$. Each object has a *size* s_1, s_2, $\ldots s_k$ and a *value* v_1, v_2, $\ldots v_k$. You have a backpack that has a size limit B and a value goal V. Find a set of objects S that fit into your backpack

$$\sum_{n_i \in S} s_i \leq B$$

whose collective value is above the value goal:

$$\sum_{n_i \in S} v_i \geq V$$

Show that *SHOPLIFTER* is $\mathcal{NP}$-complete. (Hint: consider the case where $s_i = v_i$ for all i.)

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Problem 4: **(15 points)** Show that if $\mathcal{P} = \mathcal{NP}$, there exists a polynomial time algorithm that takes in an undirected graph G and returns either

- A Hamiltonian cycle in G, if one exists, or
- A special output $\perp$ if one does not.

Note that your algorithm must return the cycle itself!

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Problem 5: **(15 points)** Recall the class of languages $co\text{-}\mathcal{NP}$:

$$L \in \mathcal{NP} \iff \overline{L} \in co\text{-}\mathcal{NP}$$

A language L is $co\text{-}\mathcal{NP}$-complete iff

1. $L \in co\text{-}\mathcal{NP}$, and
2. $\forall A \in co\text{-}\mathcal{NP}$, $A \leq_p L$.

Consider the language

$$TAUT = \{\phi \ : \ \phi \text{ is a boolean formula and } \phi \text{ is true under every assignment}\}$$

Show that $TAUT$ is $co\text{-}\mathcal{NP}$ complete. (You can assume the language

$$BOOL = \{w \ : \ w \text{ is a boolean formula } \}$$

is in $\mathcal{P}$.)