6.045J/18.400J: Automata, Computability and Complexity Prof. Ron Rivest

Handout 6: Quiz 1

For this entire quiz, $\Sigma = \{0, 1\}$.

Problem 1: True or False (3 points each)

1. A language L is "co-finite" if its complement $\Sigma^* - L$ is a finite set of strings. True or false: Every co-finite language is regular.

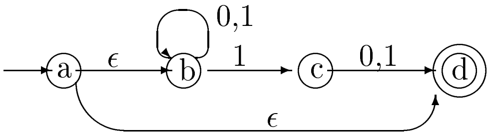
2. True or False: If a DFA is "minimal" in the sense that it has no more states than any other DFA recognizing the same language, then it must accept at least as many distinct strings as it has accepting states.

3. True or False: If L_1, L_2, and L_3 are languages such that $L_1 \subseteq L_2$ and $L_2 \subseteq L_3$ and both L_1 and L_3 are regular, then L_2 must be regular.

4. Let $Dub(L)$ be the language obtained by replacing each "0" by "00" and each "1" by "11" in each string in L. True or False: $Dub(L)$ is regular if and only if L is regular.

5. True or False: If the DFA M and the NFA M' accept the same language, then M' has no more states than M.

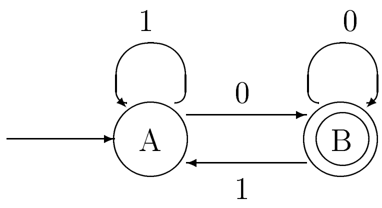
Problem 2: **(15 points)** Convert the following NFA into a DFA using the procedure we have studied in class. Your answer should be the diagram of a DFA. You may optionally explain your work if you wish.



Your DFA need only contain states reachable from the start state.

Problem 3: **(15 points)** Let R be a regular expression. Consider extending the definition of regular expressions to include the operator ^{r} (a superscript operator). If R is a regular expression, then R^r is the set of strings which, when *reversed*, match R. So if $R = 11^*0$, then R^r would describe 0111 and 01, but not 110. Show that this new operator does not extend the power of regular expressions. **In particular**, show that for every regular expression R, R^r can be rewritten algorithmically into some other equivalent regular expression R' which does not use the ^{r} operation. (If you cannot do this, you may, for partial credit, prove that regular languages are closed under reversal via a machine proof.)

Problem 4: **(15 points)** Find a regular expression for the language recognized by this machine, using the procedure we have studied in class:



Please show your work. You may omit $\emptyset$-transitions from your diagrams. Please remove state A before state B.

Problem 5: **(15 points)** Prove that the language $L = \{0^n 1^m 0^n \ : \ n, m \geq 1\}$ is not regular.