6.045J/18.400J: Automata, Computability and Complexity Prof. Ron Rivest

Homework 8

Due: 7 May 2003

Problem 1: Consider the following problem: Given a multivariate polynomial, $P(x_1, x_2, \ldots, x_n)$, determine whether the polynomial is uniformly zero when its inputs are restricted to be either 0 or 1.

Argue that a polynomial time algorithm that solves this problem is unlikely to exist.

Problem 2: Consider the problem:

$$HALF-SAT = \{\phi \quad | \quad \phi \text{is a Boolean formula that is satisfied by exactly half of the possible assignments of truth values to its variables}\}$$

Argue that HALF-SAT is unlikely to solvable in polynomial-time.

Hint: First argue that UNSAT is unlikely to solvable in polynomial time, where UNSAT is the set of boolean formulas such that no assignment of truth values to variables makes the formula true.

Problem 3: Prove that the following problem is NP-Complete:

$$NA = \{< M, x > \quad | \quad \text{M is a non-deterministic TM that accepts } x \text{ in at most } |x|^2 \text{ time steps}\}$$

Hint: Consider how running time is affected by adding zeros to the front of an algorithm's input (For correctness, the algorithm should be modified to ignore leading zeroes).