6.045J/18.400J: Automata, Computability and Complexity Prof. Ron Rivest

Homework 7

Due: 23 April 2003

Problem 1: **Polynomial-Time Reductions**

Part A: The Traveling Salesman Problem (TSP) is: Given a set of n cities, the pairwise distances between the cities and an integer k, determine if there is a tour that starts in city 1, visits each other city exactly once and ends in city 1, where the total length of the cycle is at most k. (Formally, TSP is defined as a set of pairs, $< D, k >$, such that D is an nxn matrix containing the distances between each pair of cities and the n cities defined by D contain a TSP tour of total length at most k).

The Hamiltonian Cycle Problem (HAMCYCLE) is: Given a graph, determine whether there exists a cycle (a path with the same starting and ending point) which visits each vertex exactly once. (Formally, HAMCYCLE is a set of graphs, $< G >$, that contain a Hamiltonain Cycle).

Prove that $HAMCYCLE \leq_p TSP$. (Note: Since HAMCYCLE is NP-Complete, this reduction proves the NP-Completeness of TSP).

Part B: (Sipser 7.19) The DOUBLE-SAT problem is:

$$DOUBLE{-}SAT = \{< \phi > | \phi \text{ is a boolean formula which has at least two distinct satisfying assignments}\}$$

Prove that $SAT \leq_p DOUBLE{-}SAT$ (Note: This also proves that DOUBLE-SAT is NP-Complete).

Part C: The ODD-ONES problem is:

$$ODD - ONES = \{s \in \{0,1\}^* | s \text{ contains an odd number of 1's }\}$$

Prove that $ODD{-}ONES \leq_p 3COL$ (Note: This does not prove that ODD-ONES is NP-Complete)

Problem 2: **The Cook-Levin Theorem**

Part A: (Sipser 7.32) When proving the Cook-Levin theorem, we defined a window to be a 2x3 rectangle of cells. Show that the proof would have failed if we used a 2x2 window instead. (Note: A similar argument shows that the proof would have failed if we used kx2 windows for any k).

Part B: When proving the Cook-Levin theorem, we defined a window to be a 2x3 rectangle of cells. In Part A of this problem you showed that a smaller rectangle would not have worked. In this part, we consider using a much larger rectangle of cells. Let n^kxn^k be the size of the entire tableau, show that the proof would have failed if we had used an nx3 window.