6.045J/18.400J: Automata, Computability and Complexity Prof. Ron Rivest

Homework 6

Due: 16 April 2003

Problem 1: (Sipser 7.1, 7.2) Answer each of the following with TRUE or FALSE: (Note: when dealing with sets like $O(f(n))$ or $o(f(n))$, we use the symbols $=$ and $\in$ interchangeably).

1. $2n = O(n)$
2. $n^2 = O(n)$
3. $n^3 = O(n \log^2(n))$
4. $n \log(n) = O(n^2)$
5. $3^n = 2^{O(n)}$
6. $2^{2^n} = O(2^{2^n})$
7. $n = o(2n)$
8. $2n = o(n^2)$
9. $2^n = o(3^n)$
10. $1 = o(n)$
11. $n = o(\log(n))$
12. $1 = o(1/n)$

Problem 2: (Sipser 7.6, 7.13) Prove that P is closed under each of the following operations:

1. Union
2. Concatenation
3. Complement
4. Star

Problem 3: (Sipser 7.7, 7.14) Prove that NP is closed under each of the following operations:

1. Union
2. Concatenation
3. Star

Problem 4: (Sipser 7.11) Call graphs G and H *Isomorphic* if the nodes of G can be reordered (relabeled) so that G is identical to H. Let

$$ISO = \{< G, H > | \text{G and H are Isomorphic Graphs}\}$$

Prove that $ISO \in NP$.