6.045J/18.400J: Automata, Computability and Complexity Prof. Ron Rivest

Homework 5

Due: 9 April 2003

Problem 1: Write a program (in a language other than C or C++) that prints itself twice. (Please format your solution so that it is easy to read. You are allowed to have additional whitespace in your program which is not present in the output of your program).

Problem 2: Prove that there does not exist an infinite subset of MIN_{TM} that is recognizable.

$$\begin{aligned} MIN_{TM} \quad = \quad & \{< M > | \text{There is no Turing Machine, } N\text{, such that } L(M) = L(N) \\ & \text{and the description of } N \text{ is shorter than the description of } M\} \end{aligned}$$

Problem 3: Prove the following generalization of Rice's Theorem: If P is a non-trivial property of pairs of recognizable languages, then

$$A_p = \{< M, N > | \text{M and N are Turing Machines and } P(M, N) = TRUE\}$$

is undecidable. (Note: A property, P, is *non-trivial* if there exist Turing Machines M_1, M_2, N_1 and N_2 such that $P(M_1, N_1) = FALSE$ and $P(M_2, N_2) = TRUE$).