6.045J/18.400J: Automata, Computability and Complexity Prof. Ron Rivest

Homework 3

Due: 12 March 2003

Problem 1: Show that the following language is Turing decidable:

$$L = \{cwc \quad | \quad w \text{ is a string in } \{a,b\}^* \text{ that is well-formed if a is read as a "left parenthesis" and b is read as a "right parenthesis".}\}$$

thus *cabc* and *caababbc* are in L, while *cbac* and *caabbba* is not.

One acceptable solution to this problem is to state Σ and Γ and draw a complete transition function diagram for this Turing Machine. Another acceptable solution to this problem is to state Σ and Γ and provide "pseudocode" for your procedure that is sufficiently detailed that the grader is convinced you could have written out the transition function diagram correctly.

Problem 2: Show that the collection of decidable languages is closed under the following operations: (Sipser 1.14)

1. Union
2. Concatenation
3. Star
4. Complementation
5. Intersection

Problem 3: Consider a Turing Machine whose tape is a two-dimensional array of cells with a single read/write head. (Think of it as the first quadrant of the x-y plane where each cell on the tape is identified with a pair of positive integers (x, y)). The head begins in the the lower left hand corner of the array (which we think of as the cell, $(1, 1)$) and at each step in the computation the machine can move the move the head Left, Right, Up or Down. (That is, the transition function maps $Q \times \Gamma \leftarrow Q \times \Gamma \times \{L, R, U, D\}$. It is considered a no-op if the machine attempts to move the read/write head to the left of the starting cell or below the starting cell). Prove that this machine is equivalent to a standard Turing Machine.