6.045J/18.400J: **Automata, Computability and Complexity**	Prof. Ron Rivest
Homework 0	
Due: 12 February 2003	

Note: Recitations on Thursday. 10am in 34-302. 1pm and 4pm in 34-304.

Problem 1: Construct truth tables for each of the following formulae. Additionally, for each pair of formulae, state which of the following holds:

- They are equivalent,
- They are not equivalent, but one implies the other (make sure to state which is which), or
- Neither of the above:

1. $(p \vee q) \Rightarrow (p \wedge q)$
2. $q \vee (\neg q \Rightarrow p)$
3. $(p \Rightarrow q) \wedge (q \Rightarrow p)$
4. $(\neg p \Rightarrow q) \wedge (\neg q \Rightarrow p)$

Problem 2: Let **Z** be the set of Integers. Let R be the relation on **Z** such that $\{a, b\} \in R$ if, and only if, $ab > 0$.

1. Is R Reflexive?
2. Is R Symmetric?
3. Is R Transitive?
4. Define a proper subset of R that has exactly one of the above three properties.

Problem 3: A *Hamiltonian cycle* in an undirected graph is a cycle that goes through every vertex in the graph exactly once. In this problem, we consider a special class of graphs which we will call *Happy* graphs. A Happy graph is one that has the following properties.

1. The graph has $2n$ vertices for $n \geq 2$.
2. For all $n > i \geq 1$. The graph has at least $2n - 2i + 2$ vertices of degree greater than or equal to $n + i$.

Part A: Prove that for $n = 2$, all Happy graphs with $2n$ vertices have a Hamiltonian cycle.

Part B: Let H be a Happy graph with $n \geq 3$. Prove that if you remove two vertices of minimal degree from H then the resulting graph is also Happy.

Part C: Prove that every Happy Graph has a Hamiltonian Cycle. (Hint: Use induction as well as the results from parts A and B).