6.045J/18.400J: Automata, Computability and Complexity Prof. Ron Rivest

Homework 6

Due: 10 April 2002

Problem 1:

1. Exercise 5.7
2. Problem 5.11. Provide the reduction from B to $\overline{B}$.

Problem 2: Recognizibility is linked with the idea of *unbounded existential search.* Often an algorithm for accepting a recognizable language can be characterized in terms of searching for a *witness* or *proof* that a given input x is in the language.

A binary relation R on strings is called decidable if the language

$$\{\langle x, y\rangle \ : \ R(x, y)\}$$

is decidable. Show that a language A is recognizable if and only if there exists a decidable binary relation R such that

$$A = \{x \ : \ \exists y \text{ such that } R(x, y)\}$$

Problem 3: Formalize and prove the extension of Rice's Theorem about *pairs* of recognizable languages: every non-trivial property of pairs of recognizable languages is undecidable.

Problem 4: Consider a one-tape Turing machine constrained so that it cannot overwrite the input portion of the tape. It may, however, write as much as it pleases on the blank portion to the right of the input string.

1. Prove that the language of such a Turing machine must be regular.
2. Prove that despite this, the problem of determining if such a Turing machine recognizes the same language as a given DFA is undecidable.