6.045J/18.400J: Automata, Computability and Complexity Prof. Ron Rivest

Homework 1

Due: 13 February 2002

Problem 1: Construct truth tables for all of the following formulae. For each pair of formulae, state which of the following holds:

- They are equivalent,
- They are not equivalent, but one implies the other (make sure to state which is which), or
- Neither of the above:

1. $(p \Rightarrow q) \vee (p \Rightarrow \neg q)$
2. $p \vee (p \Rightarrow q)$
3. $p \wedge (p \Rightarrow q)$
4. $(p \Rightarrow q) \vee (\neg p \Rightarrow q)$

Problem 2: Suppose R is a relation on a nonempty set A. Define $R^s \stackrel{\text{def}}{=} R \cup \{(x, y) \;:\; yRx\}$. Show that R^s is symmetric and is the smallest symmetric relation on A containing R. (We say that a set S is the smallest set that satisfies a property P if and only if for all other sets S' that satisfy P, $S \subseteq S'$.) We call R^s the *symmetric closure* of R.

Problem 3: Show that every binary tree with L leaves contains a subtree having between $L/3$ and $2L/3$ leaves, inclusive.

Problem 4: A *Hamiltonian cycle* in an undirected graph is a cycle that goes through every node in the graph exactly once. Does every connected, undirected graph, where each node has degree at least 3, contain a Hamiltonian cycle? Prove it if true, find a counterexample if false.