6.045J/18.400J: Automata, Computability and Complexity	Prof. Ron Rivest
Handout 2: Recitation Problems	
7 Wednesday 2002	*Jonathan Herzog*

Problem 1: Construct truth tables for all of the following formulae. For each pair of formulae, state which of the following holds:

- They are equivalent,
- They are not equivalent, but one implies the other (make sure to state which is which), or
- Neither of the above:

1. $p \Rightarrow q$
2. $p \vee q$
3. $p \wedge q$
4. $\neg p \vee q$
5. $\neg(\neg p \vee \neg q)$

Problem 2: A Boolean formula is in *disjunctive normal form* when it is in the form $(x_1 \wedge \neg x_2 \wedge x_3 \ldots) \vee (x'_1 \wedge x'_2 \wedge \neg x'_3 \ldots) \vee \ldots$

1. Find a boolean formula in disjunctive normal form equivalent to:
   (a) $(p \wedge q) \Rightarrow r$
   (b) $(p \vee q) \Rightarrow r$
2. Sketch a proof that any Boolean formula has an equivalent formula in disjunctive normal form.

Problem 3: Suppose R is a relation on a nonempty set A. Define R^t to be the intersection of all transitive relations on A that contain R. Show that R^t is transitive and the smallest transitive relation on A containing R.

Problem 4: Let $R^u \stackrel{\text{def}}{=} R \cup \{(x, y) \ : \ \exists z.(xRz \wedge zRy)\}$. Is $R^u = R^t$ from above? Prove that it is, or find a counterexample.

Problem 5: In a binary tree, what is the relationship between the number of internal nodes and the number of leaves? Prove your answer.

Problem 6: Create a finite automata, like the kind seen in class, that accepts the language $L_i = \{x \ : \ x \text{ as a binary number is a multiple of } i\}$, where

1. $i = 2$,
2. $i = 4$,
3. $i = 6$