6.045J/18.400J: Automata, Computability and Complexity Prof. Ron Rivest

Handout 18: Recitation Problems

9 May 2002 *Jonathan Herzog*

1. Suppose L is a language and M is a probabilistic polynomial-time decider such that
   - $Pr\left[M(x) \text{ accepts } | x \in L\right] \geq .9$, and
   - $Pr\left[M(x) \text{ accepts } | x \notin L\right] \leq .1$

   Is $L \in \mathcal{BPP}$?
2. Suppose L is a language and M is a probabilistic polynomial-time decider such that
   - $Pr\left[M(x) \text{ accepts } | x \in L\right] \geq .6$, and
   - $Pr\left[M(x) \text{ accepts } | x \notin L\right] \leq .4$

   Is $L \in \mathcal{BPP}$?
3. Suppose L is a language and M is a probabilistic polynomial-time decider such that
   - $Pr\left[M(x) \text{ accepts } | x \in L\right] \geq .9$, and
   - $Pr\left[M(x) \text{ accepts } | x \notin L\right] \leq .8$

   Is $L \in \mathcal{BPP}$?
4. Suppose L is a language and M is a probabilistic polynomial-time decider such that
   - $Pr\left[M(x) \text{ accepts } | x \in L\right] \geq .1$, and
   - $Pr\left[M(x) \text{ accepts } | x \notin L\right] = 0$.

   Is $L \in \mathcal{BPP}$? Is $L \in \mathcal{NP}$?
5. Suppose L is a language and M is a probabilistic polynomial-time decider such that
   - $Pr\left[M(x) \text{ accepts } | x \in L\right] = \frac{1}{2^{|x|}}$, and
   - $Pr\left[M(x) \text{ accepts } | x \notin L\right] = 0$.

   Is $L \in \mathcal{BPP}$? Is $L \in \mathcal{NP}$?
6. Suppose L is a language and M is a probabilistic polynomial-time decider such that
   - $Pr\left[M(x) \text{ accepts } | x \in L\right] = 1 - \frac{1}{2^{|x|}}$, and
   - $Pr\left[M(x) \text{ accepts } | x \notin L\right] = \frac{1}{2^{|x|}}$.

   Is $L \in \mathcal{BPP}$? Is $L \in \mathcal{NP}$?
7. Suppose you have a source of unbiased bits. Show how to pick a random element from $\{1, \ldots n\}$ with uniform probability. How many bits do you expect to use?
8. Suppose you have a source of unbiased bits. Show how to pick a random element from $\{2, 3, \ldots 12\}$ with the same probability as that of rolling two dice. How many bits do you expect to use?
9. Suppose you have a source of unbiased bits. Show how to pick a random permutation on $\{1, 2, \ldots n\}$ with uniform probability. How many bits do you expect to use?