6.045J/18.400J: Automata, Computability and Complexity Prof. Ron Rivest

Handout 14: Recitation Problems

18 April 2002 *Jonathan Herzog*

Which of the following are valid?

1. Let
$$SAT = \{\phi \ : \ \phi \text{ has a satisfying assignment } \}$$
and
$$DoubleSAT = \{\phi \ : \ \phi \text{ has at least two satisfying assignments } \}.$$
Consider the function from boolean formulae to boolean formulae given by:
$$t(\phi) = \phi \wedge (x \vee \overline{x})$$
where x is a variable not otherwise occurring in ϕ.
The function t is a mapping from SAT to $DoubleSAT$ and so
$$SAT \leq_p DoubleSAT$$

2. Let
$$TRIANGLE = \{G \ : \ G \text{ has a clique of size } 3\}$$
and
$$HAMCYCLE = \{G \ : \ G \text{ has a Hamiltonian cycle } \}.$$
Consider the function from graphs to graphs given by:
$$t(G) = \begin{cases} \text{if } G \text{ has a triangle, that triangle} \\ \text{if } G \text{ does not have a triangle, the sub-graph of } G \text{ formed by its first 3 nodes} \end{cases}$$
The function t is a mapping from $TRIANGLE$ to $HAMCYCLE$ and so
$$TRIANGLE \leq_p HAMCYCLE$$

3. Consider the function from boolean formulae to graphs given by:
$$t(\phi) = \begin{cases} \text{if } \phi \text{ has a satisfying assignment, a triangle} \\ \text{if } \phi \text{ has no satisfying assignment, a triangle missing one edge} \end{cases}$$
The function t is a mapping from SAT to $TRIANGLE$ and so
$$SAT \leq_p TRIANGLE$$